
Inertial System Requirements for Deflection of the Vertical Compensations

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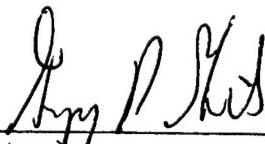
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14. ABSTRACT The object of this study was to produce information on the implementation details required to produce an inertial system suitable for gravity gradient measurements in a hypervelocity vehicle. The approach taken was to develop a deterministic model of an inertial system. Using actual hypervelocity trajectories, a gravity deflection signal was produced and passed through the inertial navigation model. Implementation errors such as time latencies and quantization were then compared to the velocity/position error produced by the gravity errors.						
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Introduction

This is the final report on a study to evaluate the requirements for an inertial system to reduce gravity-induced errors during hypersonic flight. GPS has become very widely used for precise long-term navigation and in many situations it is very effective. It does, however, have some weaknesses. The signal from the GPS satellites can be interrupted relatively easily. This interruption can be from a deliberate external source jamming the GPS Signal. Environmental effects can also obstruct (or block) the signal. The potential also exists during a period of international tension that a foreign power could disable the GPS satellites themselves.

The requirement for precise navigation for a hypervelocity vehicle produces its own set of additional constraints. The ablation/plume environment could prevent the GPS signal from being received. The higher velocity also increases the distance between any GPS updates (if available at all). These factors suggest the need for an accurate autonomous navigation system. The long-term accuracy obtainable from an Inertial Navigation System (INS) is limited by its knowledge of gravity. To advance the long-term accuracy of an inertial navigation system the gravity induced errors must be addressed. More accurate gravity maps would be required. The combination of a mobile gravity gradiometer and an accurate inertial system could be used to produce a gravity map referenced system.

Conventional covariance simulations of an inertial system will produce a specification for the inertial components required for gravity vector measurements. Unfortunately the limiting factors in the performance of the inertial system in this application are usually the details of the implementation. These include accuracy of the time synchronization and the quantization of the available data. Earlier in this project an LN-93 navigation system was being evaluated for its suitability for gravity mapping. Although the inertial sensors themselves were sufficiently accurate the data available to the user was not usable in this application due to the time tagging and data quantization issues. A hypervelocity vehicle increases the sensitivity to some of these effects. This study aims to provide information on the implementation details required to produce a inertial system suitable for gravity gradient measurements in a hypervelocity vehicle.

Approach

The approach taken was to develop a deterministic model of an inertial system. Using actual hypervelocity trajectories a gravity deflection signal was produced and passed through the inertial navigation model. Implementation errors such as time latencies and quantization were then compared to the velocity / position error produced by the gravity errors. For the INS to be useful in this application the errors produced by the implementation must be small compared with the gravity error induced signal.

Trajectory Generation

Hypervelocity trajectories were provided as examples by the sponsor. Two trajectories were supplied representing a "ballistic missile" trajectories. These two trajectories will be identified simply by the start and finish points of the trajectories.

Trajectory #1 =	Vandenberg	-	Hickam
Trajectory #2 =	Florida Keys	-	Eglin

Paper copies of the trajectories were provided. To make this data usable in the simulation a series of individual points were obtained from the trajectory and then cubic splines fit to these points. The resulting

functions were then plotted and compared with the original trajectories. The trajectory information used for the simulations is provided in the attached plots labeled TRAJ1_1.. and TRAJ2_1..

Inertial Navigation System Model

The inertial system error model was produced in a block diagram simulation environment. This environment is called SIMULINK and is an option in the very popular MATLAB analysis environment. MATLAB provided "industry standard" mathematical analysis and plotting tools. SIMULINK provides an environment in which a mathematical model can be developed and simulated. The block diagram approach is a very effective way of documenting and explaining a complex model. Many scientists and engineers prefer this environment to the alternative of pages and pages of computer listings (e.g. C++, or FORTRAN).

In an attempt to make the model more easily understood by people not directly involved in its development it was decided to use the notation and base the model on the widely used text by Kenneth R. Britting (see Ref 1). The model consists of a reference (or truth) solution as well as the model INS system including inertial sensor errors as well as gravity anomalies.

Gravity Error model

Filter white noise is used to represent the gravity errors. The correlation distance of the noise is taken as 20 nautical miles. The gravity errors are a spatial effect to use this information in the simulation the error must be converted to a temporal signal. This is achieved by making the bandwidth of the filter a function of the velocity. Adjustments to the filter are made to ensure that the RMS of the gravity errors remains constant. The RMS of the gravity error is chosen to be 10 microg. A test was also performed at 1 microg to show that the results were relatively linear and could be scaled to the required error. The definition of the gravity noise filter and velocity dependence is shown in attached A2. A plot of the gravity filter time constant for the two trajectories is shown in figure A5.

Reference Simulations

The methodology used was to simulate the two trajectories using three difference noise seeds for the gravity errors. The results from these tests are shown in pages REF_1... These plots show the velocity errors in the North and East axis etc. with the only error being the gravity errors.

Ref_1 ..8 for gravity error seed # 1
Ref_9...17 for gravity error seed # 2
Ref_18...24 for gravity error seed # 3

The effect of reducing the magnitude of the RMS gravity error to 1 microg is shown in REF_26. This demonstrates that the result as essentially linear.

Examination of the North/East vertical deflections and the gravity anomaly (e.g. REF_1) show the effect of changing the gravity filter time constant as a function of velocity.

The purpose of the reference run is to determine the magnitude of the gravity errors. This in turn indicates the level of accuracy required in the rest of the IMS. If the level of (say) 10 microg's is required then the

implementation errors need to be one 10th of this value. This gives a quantitative value to compare the implementation errors to.

Due to the nature of the gravity errors they integrate up to significant errors and this tends to obscure the detail (or fine structure of the error signature). It is the fine structure of the error that is of interest for this study. The long term errors were removed by extracting a 4th order fit from the data leaving the residual short term error signature.

Implementation Error Sources

Two very important and quite often overlooked errors are the accuracy of time tagging the data and the quantization of the available data. The first of these errors studied was the accuracy of the data time tagging. Known latencies in the data transmission paths do not significantly degrade performance but uncertainties in the time validity of the data have very large effects.

Time Tagging

The simulations were performed with a timing error of 1 msec. This time is comparable to the least significant bit of the clock on a MIL STD 1553 bus (a common avionics interface). The errors are shown in figure RES_1 to 4. These show that a value of 1 msec is far too large and that a value of less than 10 microsec is probably required.

Data Quantization

A detailed explanation of how the velocity output quantization was computed is shown in attachment A4. This expression was used to evaluate the requirement during the two trajectories. The results are plotted in figure RES_5 (for a 10 microg error). The results indicate that a quantization of less than 0.2 mm/sec would be required.

Summary

The implementation errors studied indicate a specification of less than 10 microseconds time tagging error and less than 0.2 mm/sec velocity output data quantization.

GRAVITY ERROR RUNS TRAJECTORIES, FILES, AND PROCEDURES

Generation of gravity errors and their integration to position errors

1. Load trajectory by typing:

mg_fk for Florida Keys trajectory

mg_vand for Vandenberg trajectory

2. Run mg_grav_a

This model generates three axes of gravity errors with $\sigma=1$ by filtering white noise through a filter with velocity dependent gains. The gains are adjusted to give a correlation distance $D=20\text{-NM}$.

3. Type make_grav_a

This plots the gravity errors, scales them to $\sigma=10\text{-}\mu\text{g}$, and assembles them into three arrays with time in the first column for use in mg1. (Edit make_grav_a.m to insert trajectory name, etc. into the title of the first plot)

4. Run mg1

This model adds the gravity errors to the specific force.

5. Type plot_err

This plots North and East position and tilt errors, and North and East velocity errors. (Edit plot_err.m to insert trajectory name, etc. into the title of the first plot)

6. Type hi_pass to plot velocity errors with low frequency component removed.

Bandlimited white noise seeds used in gravity runs:

	N	E	D
#1	23341	34489	67321
#2	89273	62071	11820
#3	3708	4819	5183

Velocity error statistics from simulation runs (in m/sec):

#1 - FK Trajectory

	N	E
rms	.0029	.0050
max	.0086	.0133
min	-.0068	-.0104

#2 - FK Trajectory

	N	E
rms	.0039	.0037
max	.0093	.0084
min	-.0053	-.0110

#3 - FK Trajectory

	N	E
rms	.0033	.0040
max	.0083	.0073
min	-.0059	-.0133

#1 - VAND Trajectory

	N	E
rms	.0030	.0036
max	.0093	.0098
min	-.0066	-.0080

#2 - VAND Trajectory

	N	E
rms	.0049	.0035
max	.0110	.0075
min	-.0115	-.0082

#3 - VAND Trajectory

	N	E
rms	.0031	.0031
max	.0076	.0060
min	-.0065	-.0107

Gravity error filter

Ref: Heller, W.G., "Free Inertial and Damped Inertial Navigation Mechanization and Error Equations," Analytic Sciences Corp., Reading MA, Rept. No. TR-312-1-1, Oct. 1974.

In the reference, a model of gravity errors is constructed by filtering white noise as follows:

$$G_x = \frac{2\sqrt{2}\cdot\beta^{\frac{3}{2}}}{(s+\beta)^2} \cdot w$$

where

w is the white noise input

$$\beta = 2.146 \cdot \frac{V}{D}$$

V is the ground speed

D=20 NM is the correlation distance

The numerator factor $\beta^{\frac{3}{2}}$ is chosen so that the rms of G_x will be independent of ground speed. This can be seen by showing that the noise equivalent bandwidth is independent of β :

$$B_L \left[\frac{2\sqrt{2}\cdot\beta^{\frac{3}{2}}}{(s+\beta)^2} \right] = 1$$

The discrete version of this filter used in the simulation is:

$$H(z) = (1 - R^2)^{\frac{3}{2}} \cdot \left(\frac{z^{-1}}{1 - R \cdot z^{-1}} \right)^2$$

where $R = e^{-\beta \cdot dt}$ and dt is the sample time. It can be shown that:

$$B_L(H(z)) = \frac{1}{2} \cdot (1 + R^2)$$

which is approximately equal to 1 if the sample time is such that $\beta \cdot dt \ll 1$.

Time tag errors

1. Load trajectory by typing:

mg_fk for Florida Keys trajectory

mg_vand for Vandenberg trajectory

2. Run mg0

This integrates the trajectory data to give the complete nav solution

3. Type time_tag

This performs a first difference on N and E position and velocity, and scales the results to a 1 msec time tag error. The results are plotted.

Output Quantization Error

If the specific force error is computed using the first difference of INS velocity, this computation must be carried out at least 10 times per gravity error time constant in order to be useful:

$$A_n = \frac{V_n - V_{n-1}}{0.1 \cdot \tau}$$

The error in this computation due to velocity output quantization error is:

$$\delta A_n = \frac{\delta V_n - \delta V_{n-1}}{0.1 \cdot \tau}$$

which gives $\sigma_{\delta A_n}^2 = 2 \cdot \frac{\sigma_v^2}{(0.1 \cdot \tau)^2}$ or $\sigma_{\delta A_n} = \sqrt{2} \cdot \frac{\sigma_v}{0.1 \cdot \tau}$

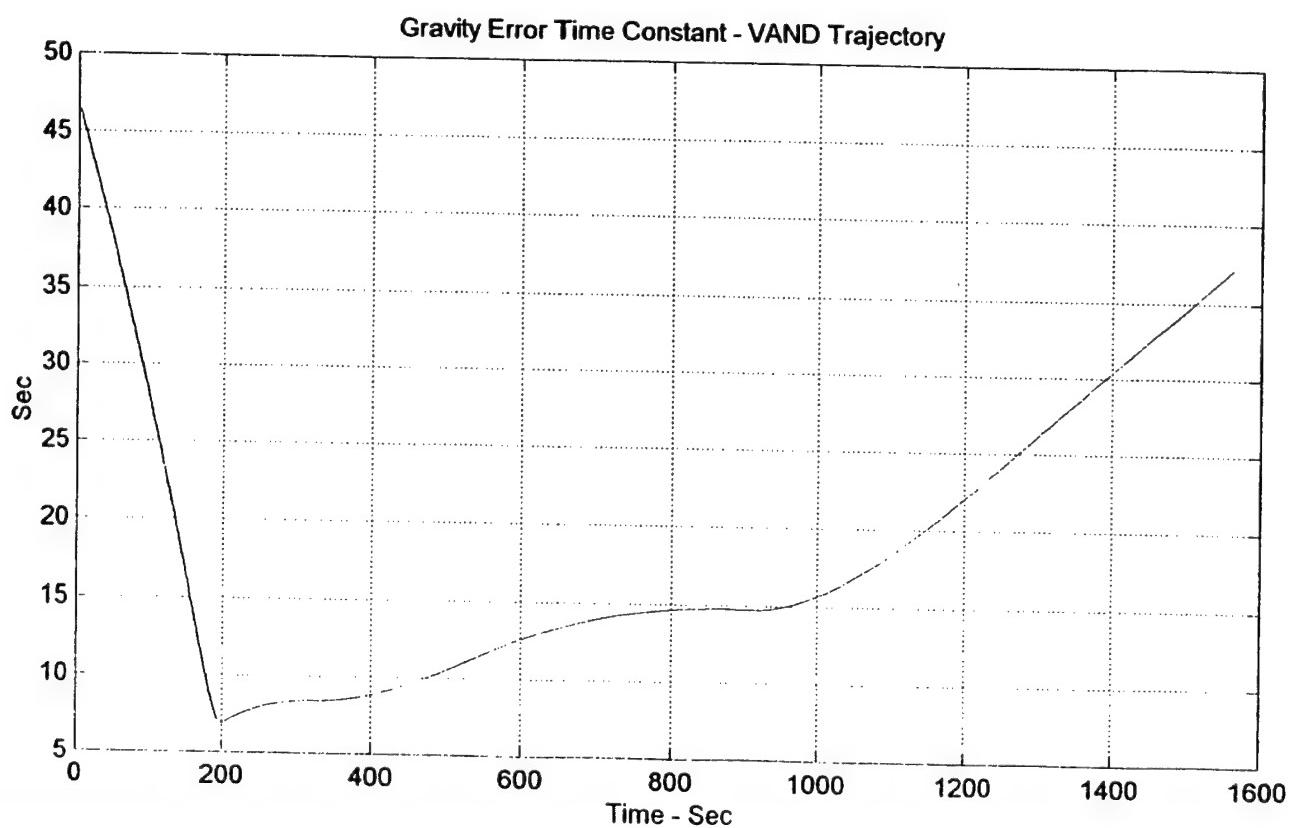
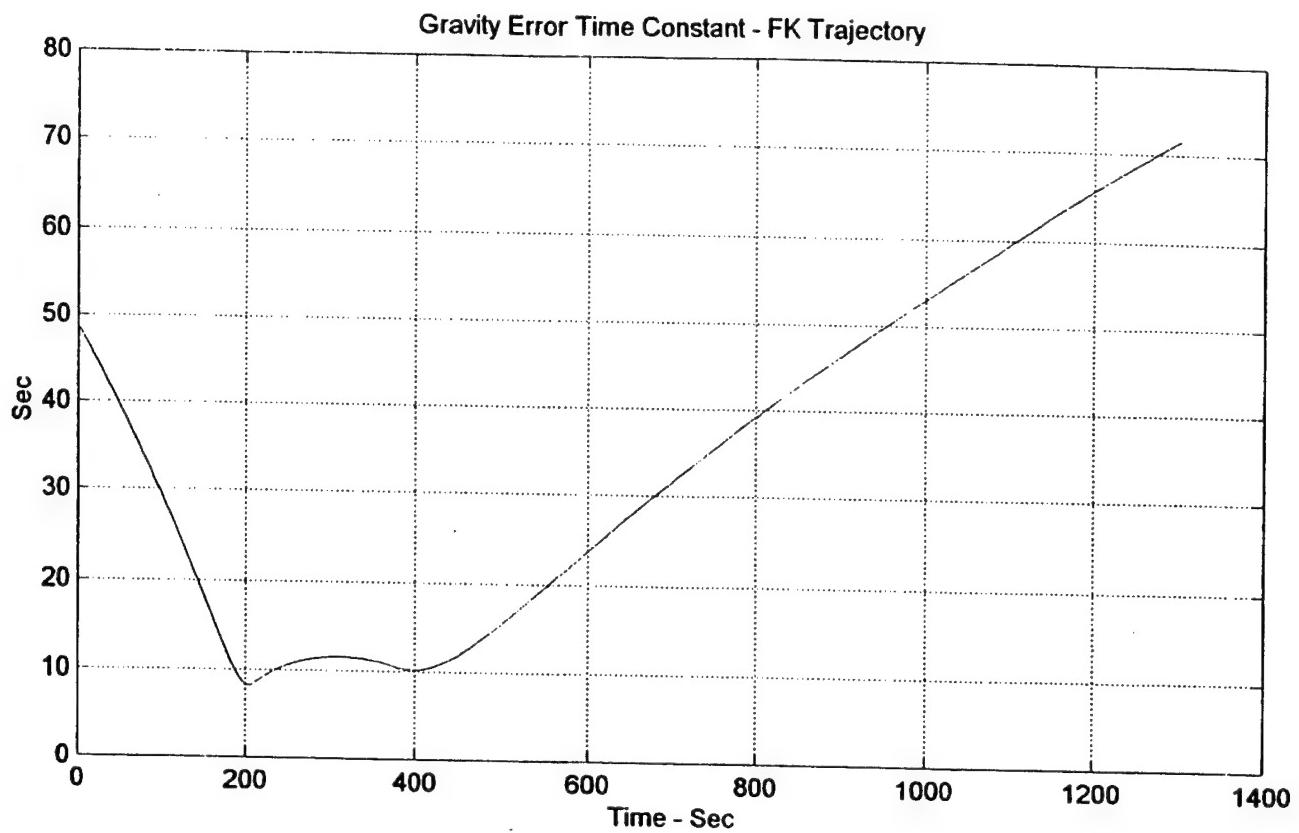
But $\tau = \frac{D}{V}$, and the velocity quantization, Q_V , gives $\sigma_v = \frac{Q_V}{\sqrt{12}}$, so

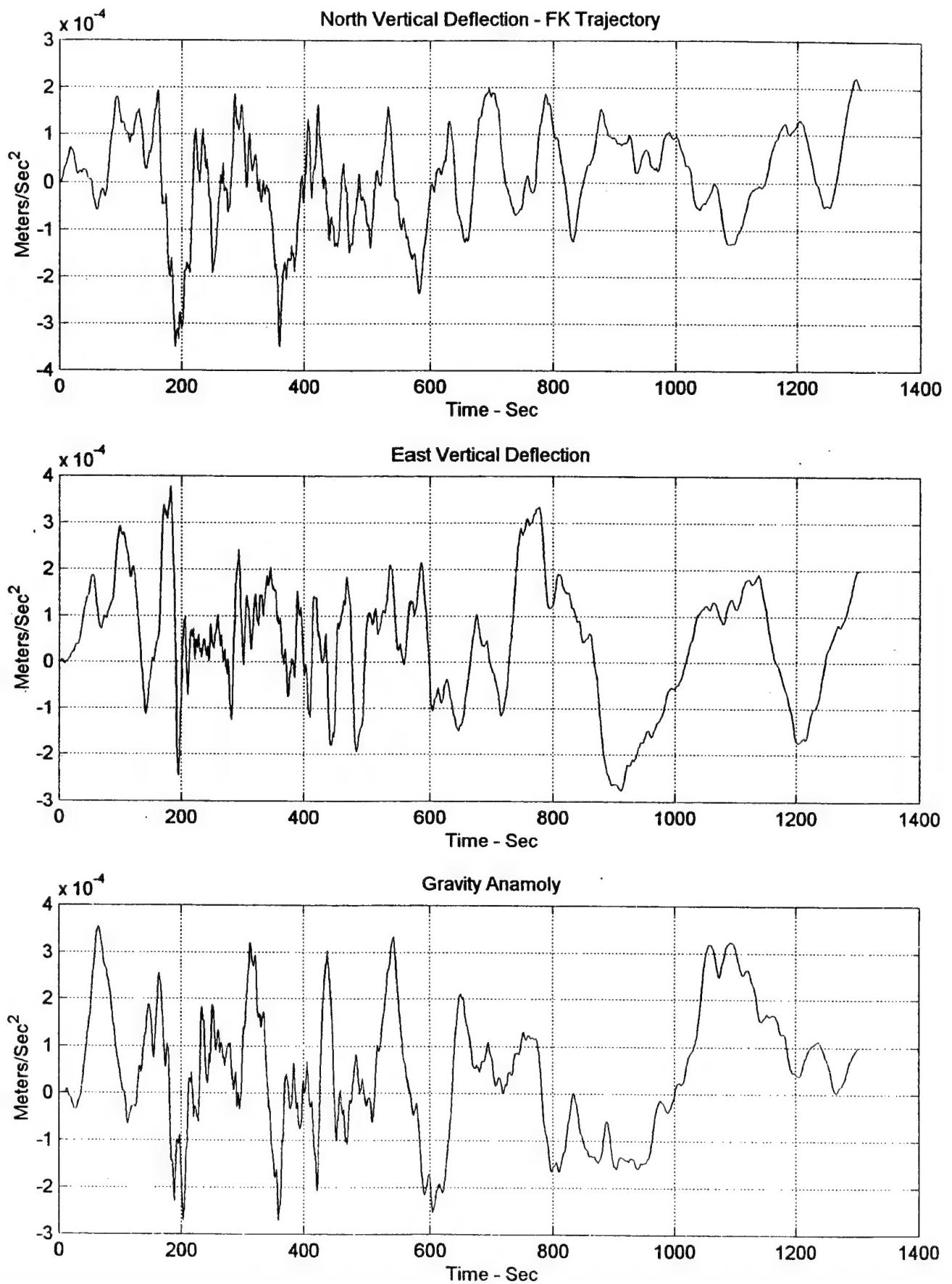
$$\sigma_{\delta A_n} = \sqrt{2} \cdot \frac{\frac{Q_V}{\sqrt{12}}}{0.1 \cdot \frac{D}{V}}$$

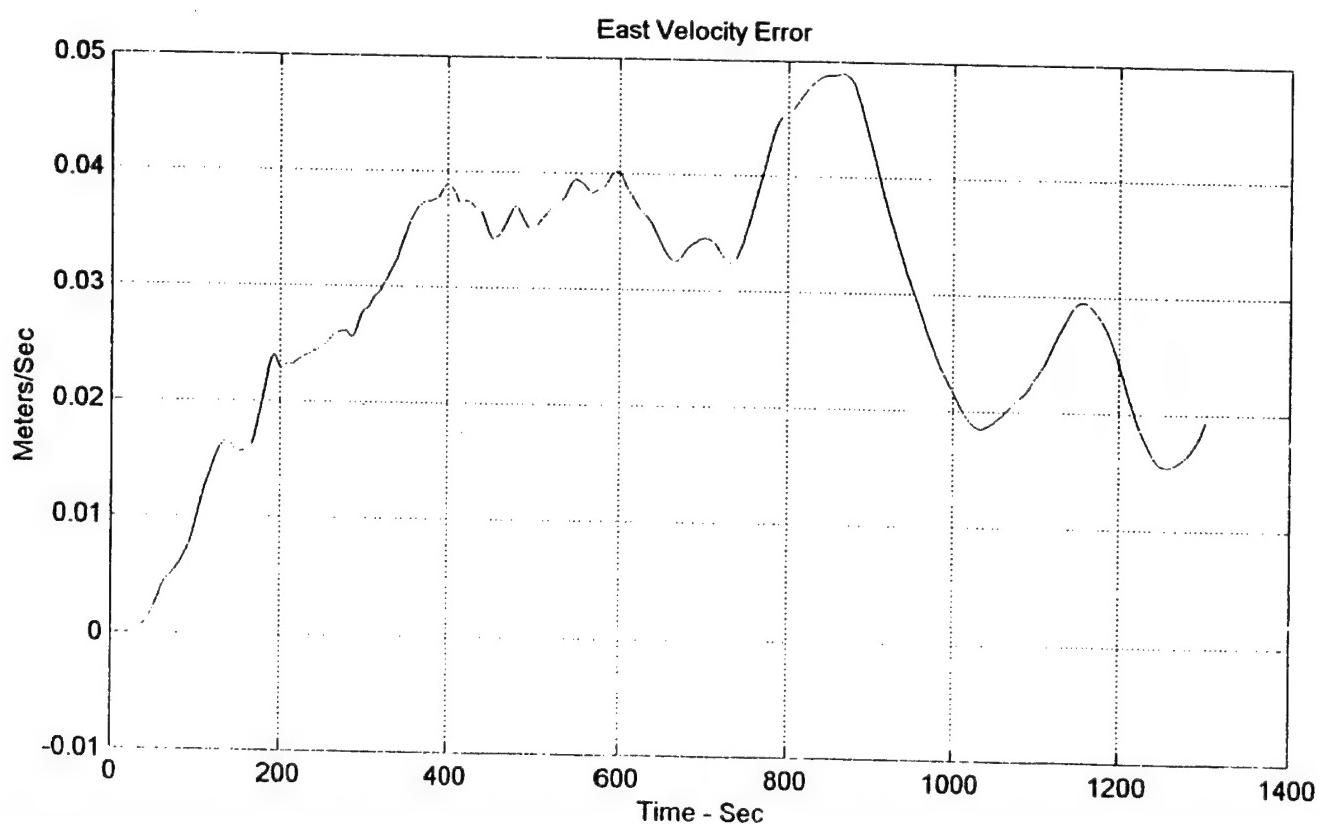
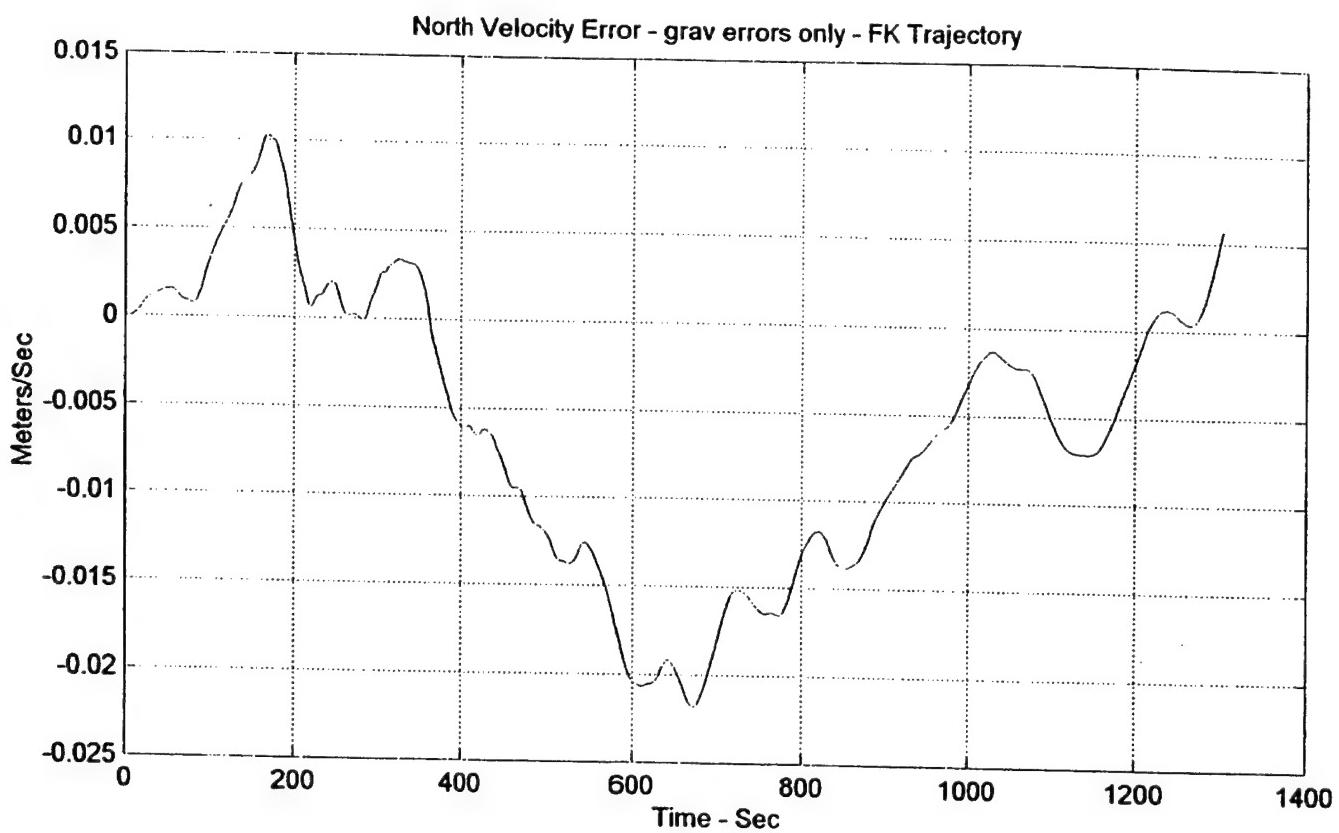
Solving for Q_V gives: $Q_V = \frac{\sqrt{6} \cdot D}{10 \cdot V} \cdot \sigma_{\delta A_n}$

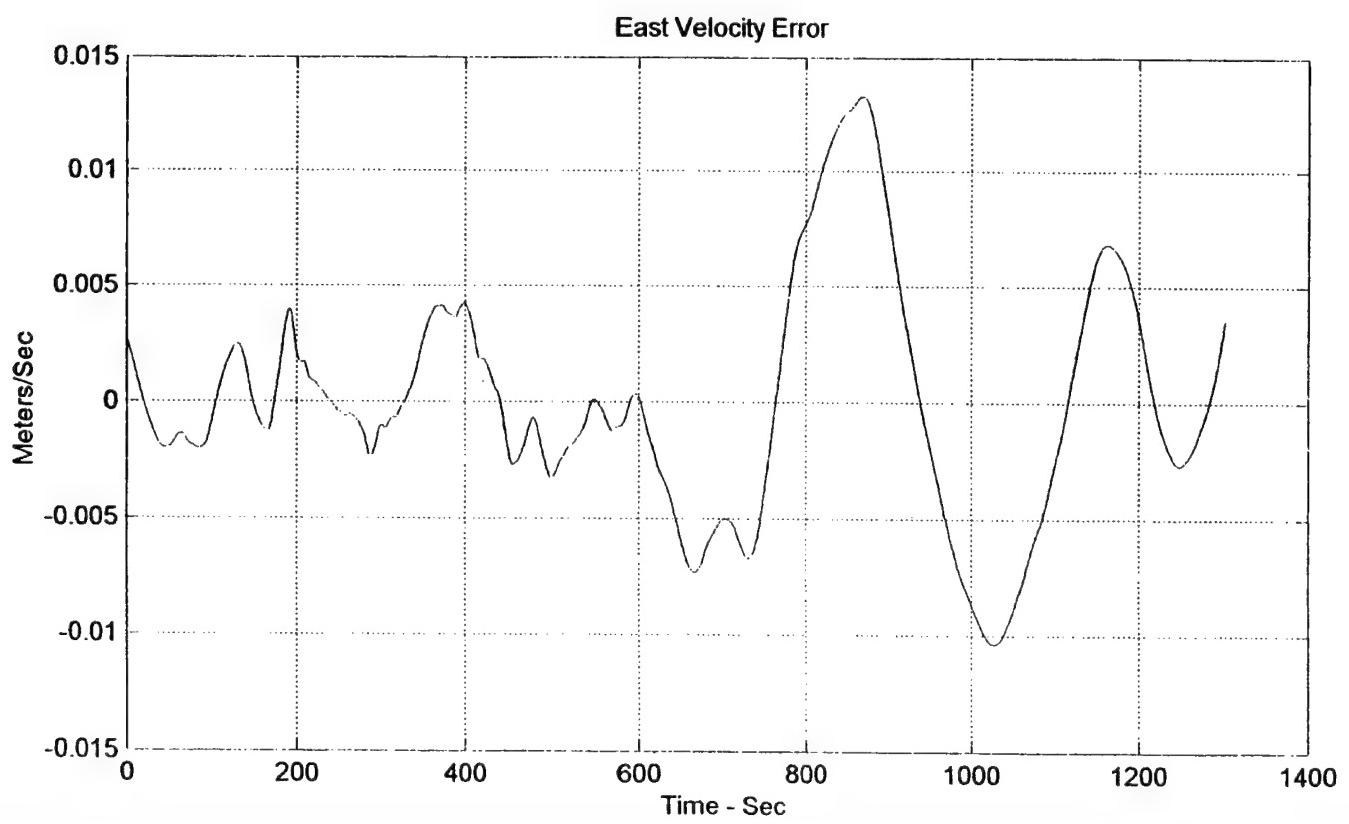
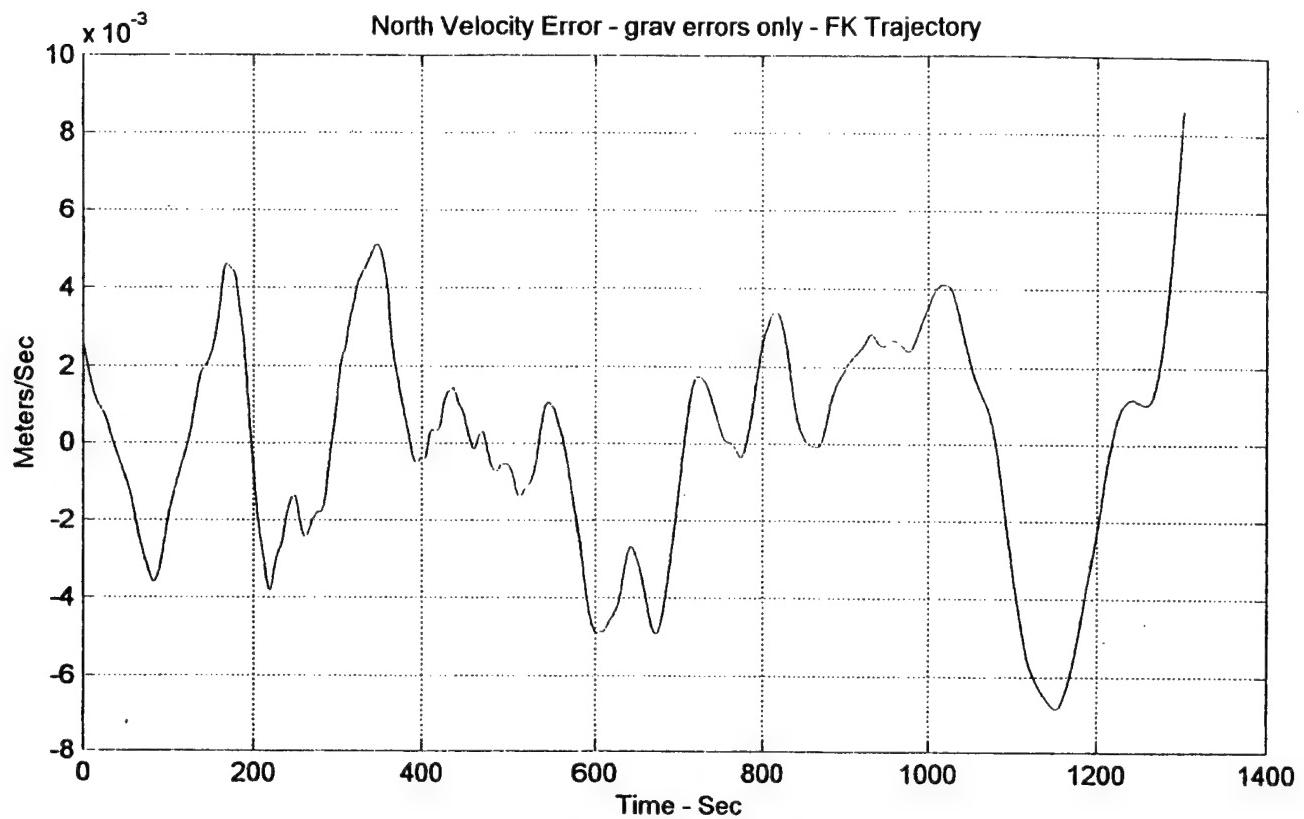
Velocity Output LSB Requirement Plot

1. Load trajectory by typing:
`mg_fk` for Florida Keys trajectory
`mg_vand` for Vandenberg trajectory
2. Run `mg0`
3. Type `plot_Q`

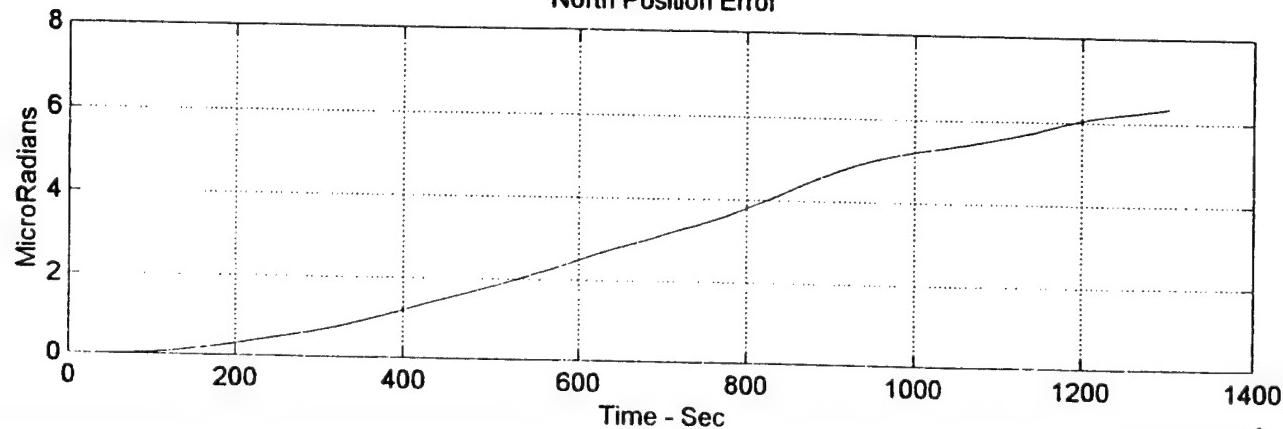
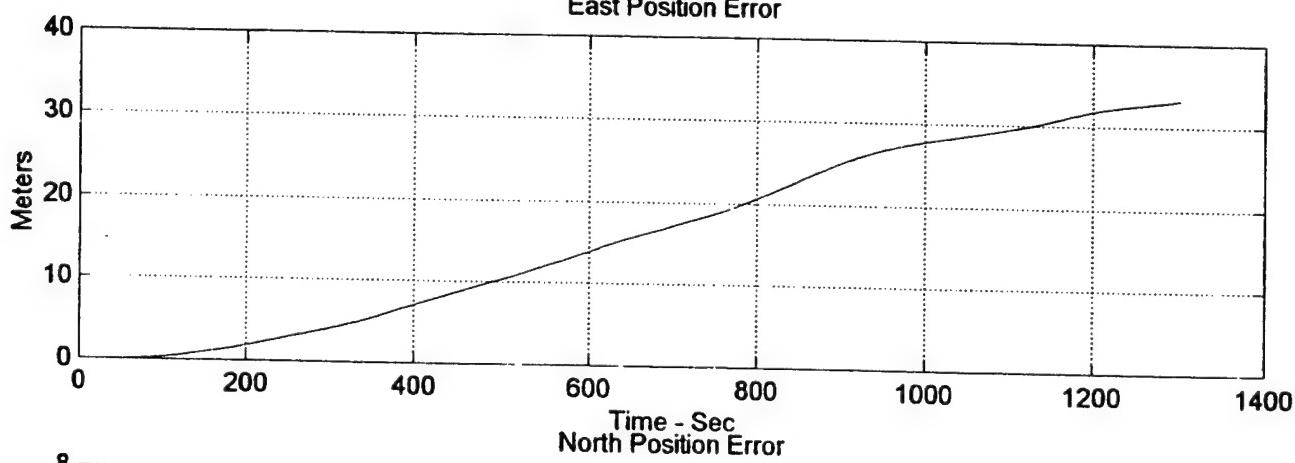
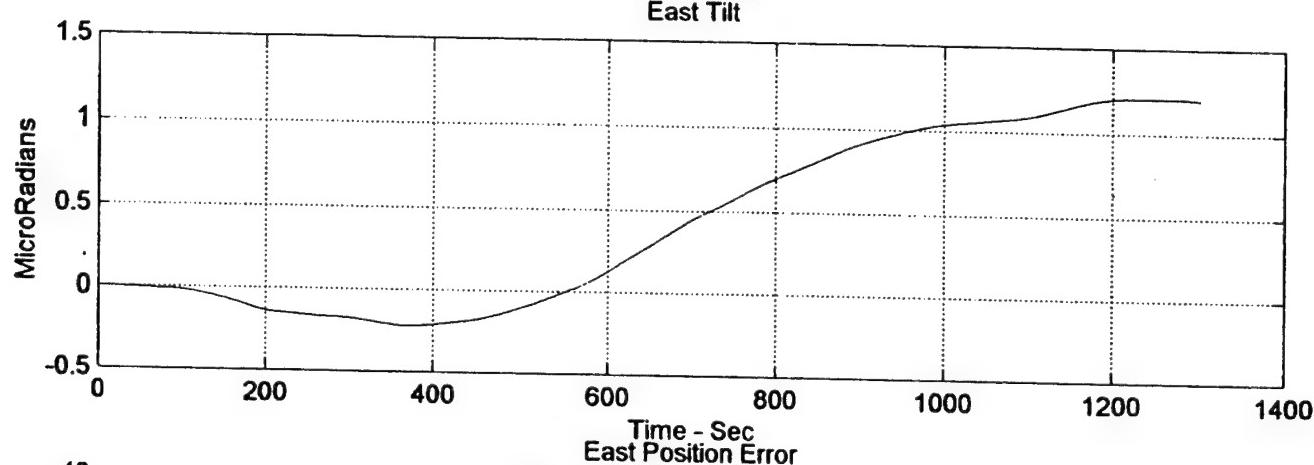
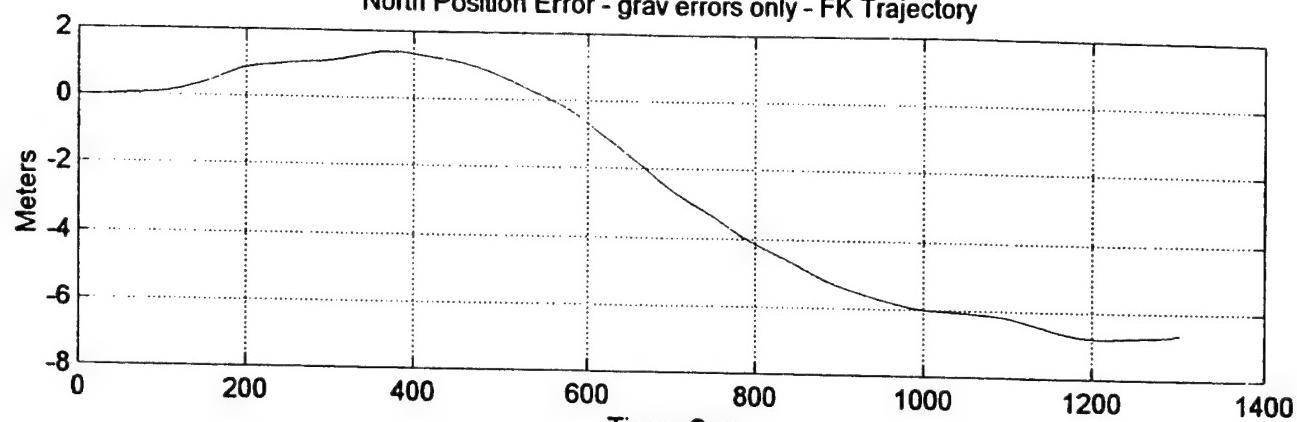


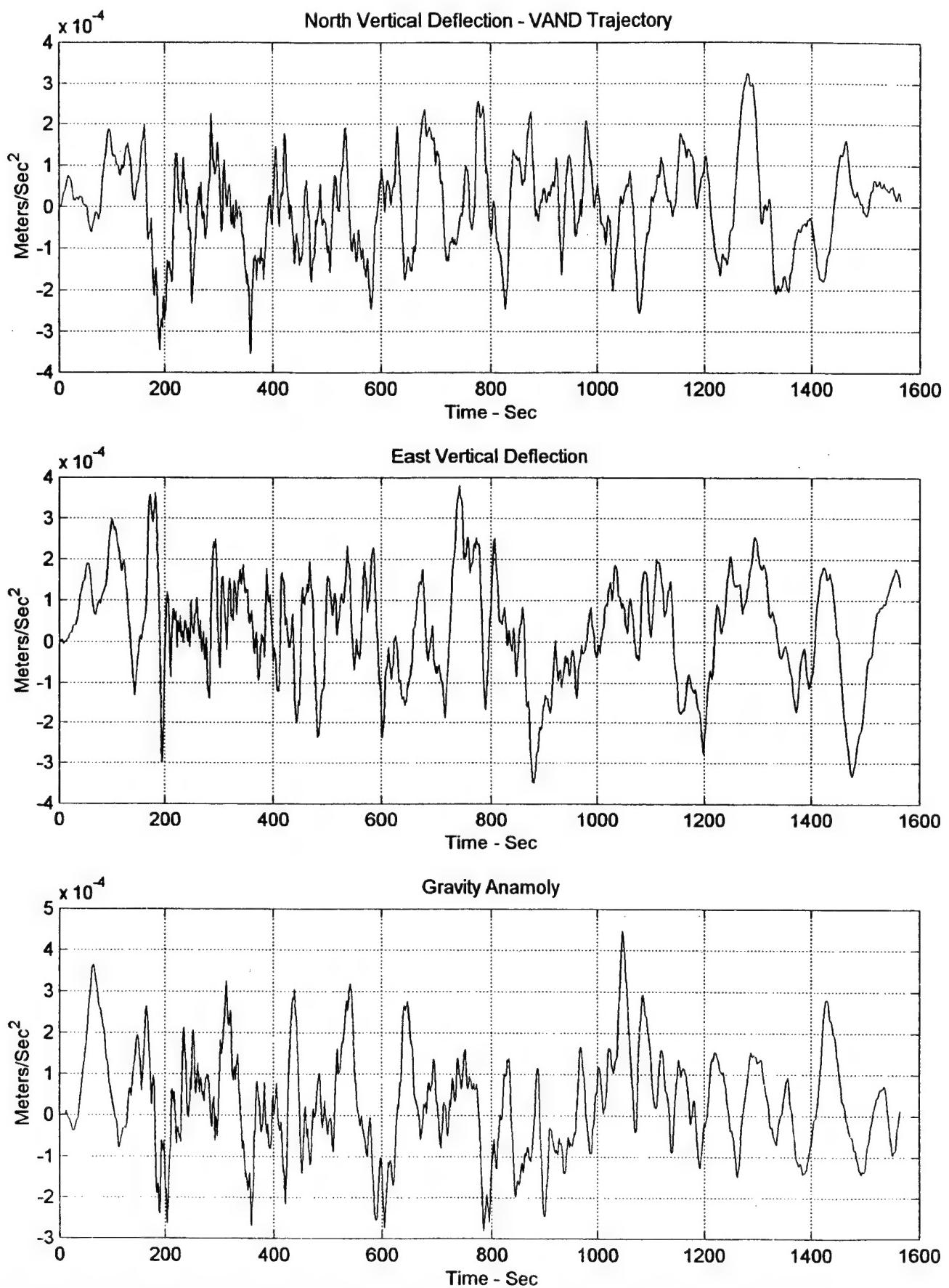




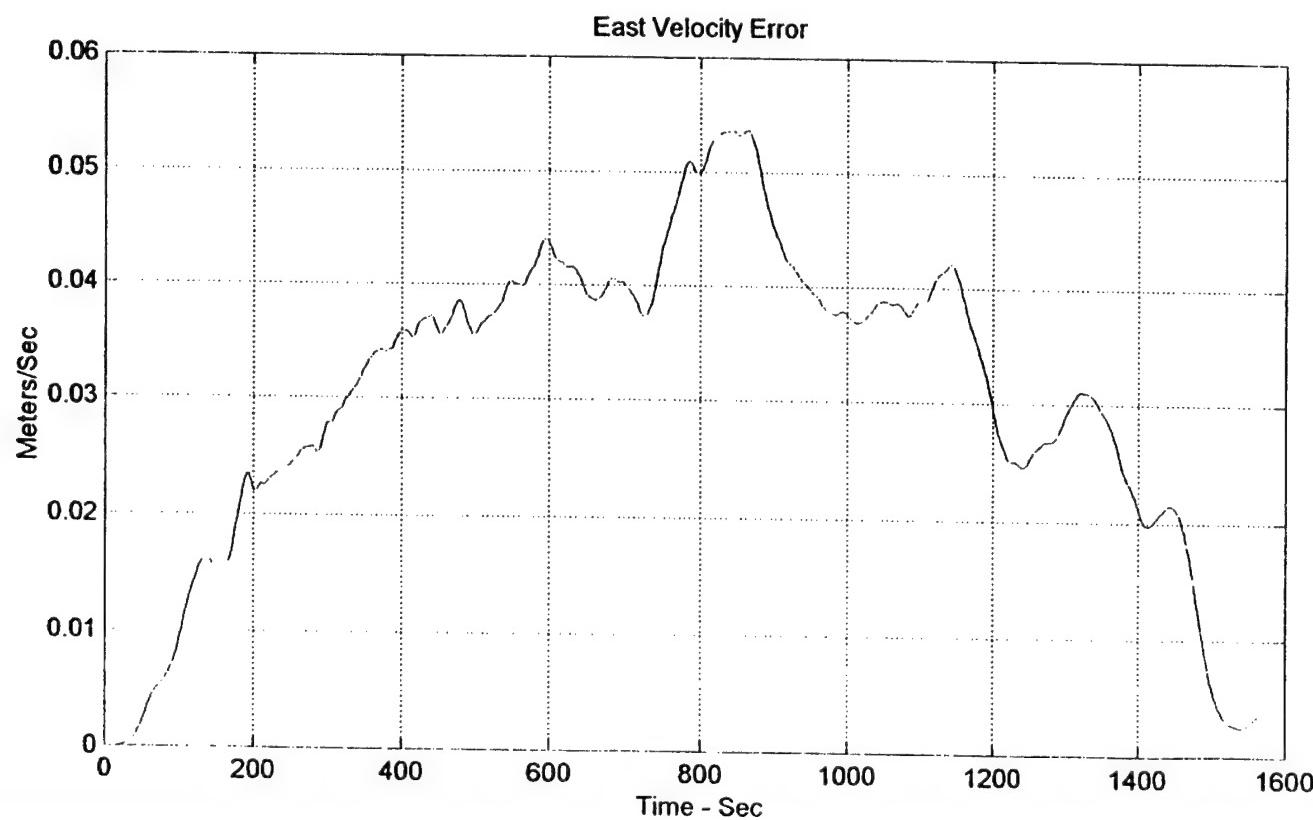
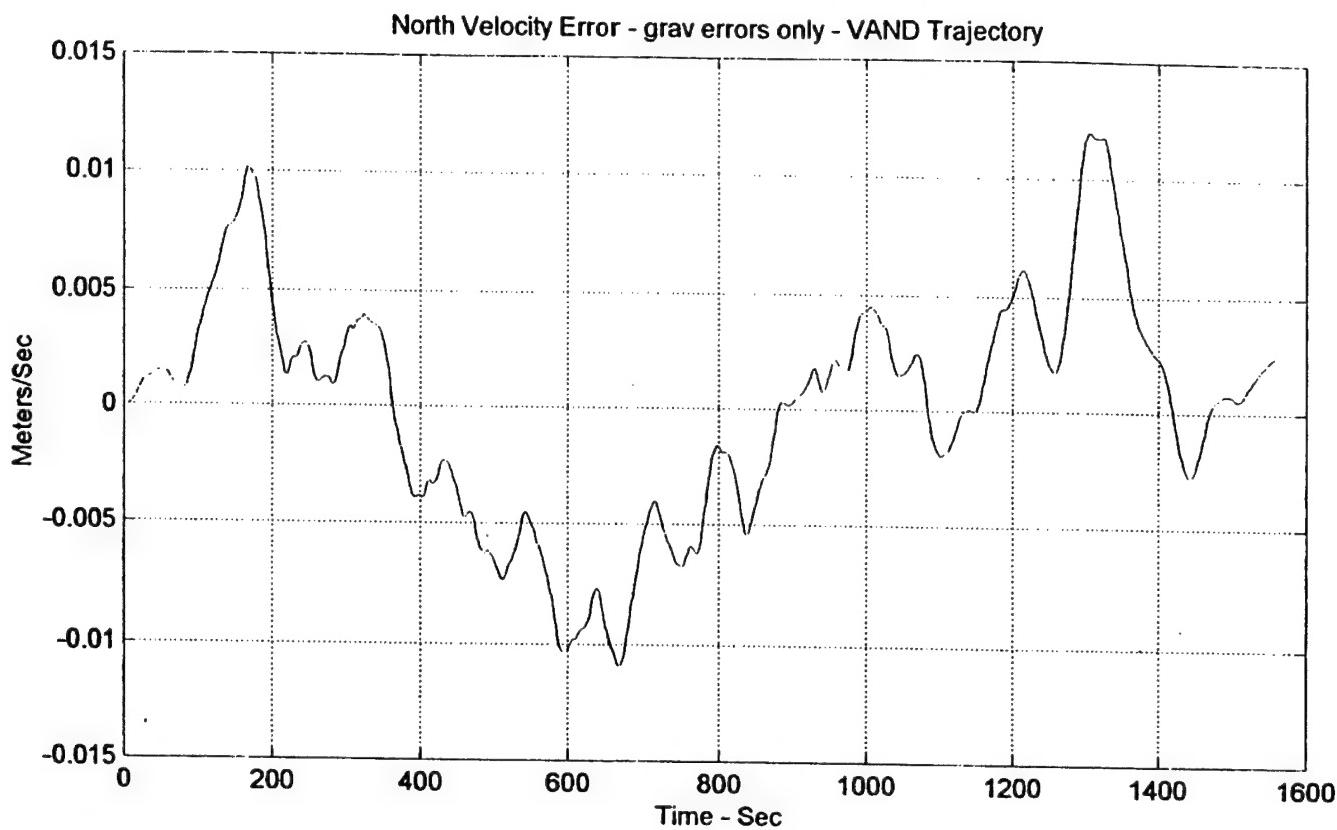


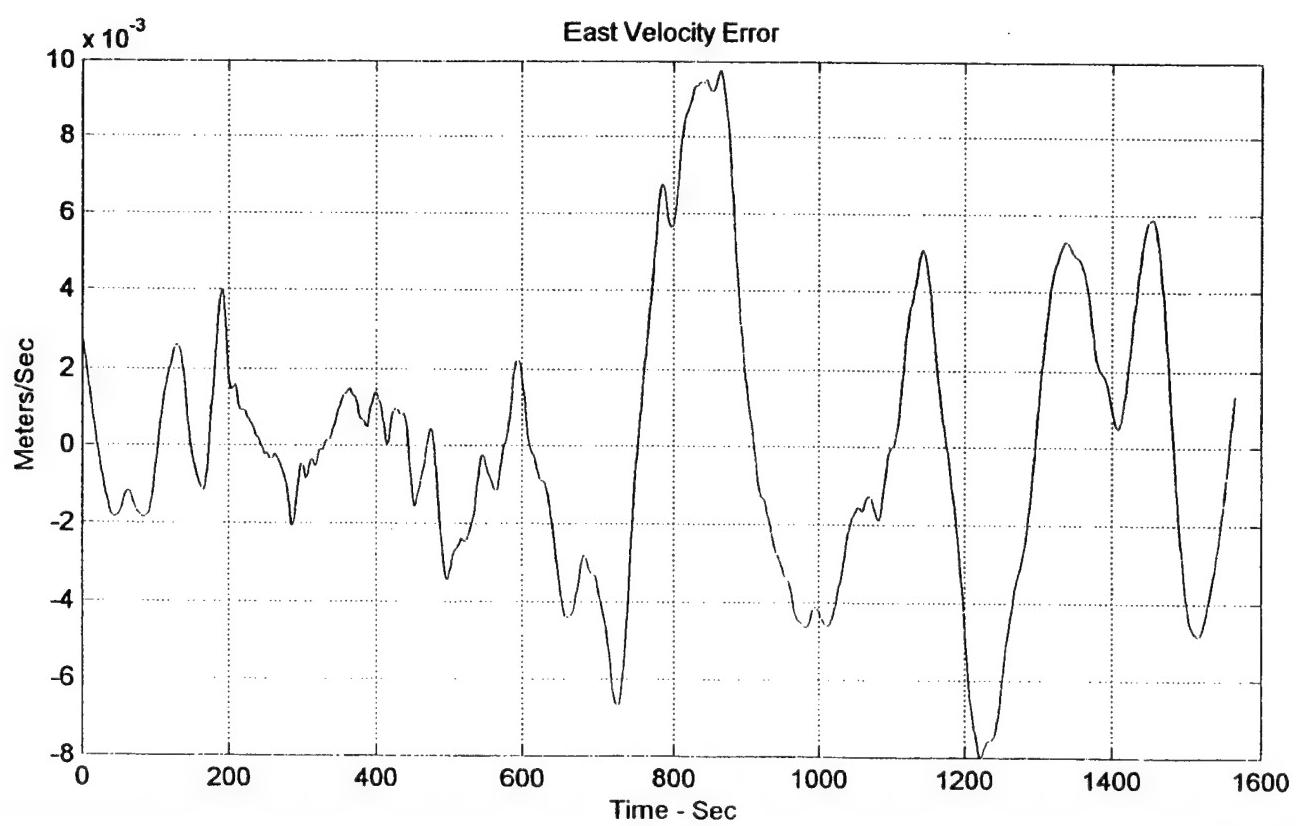
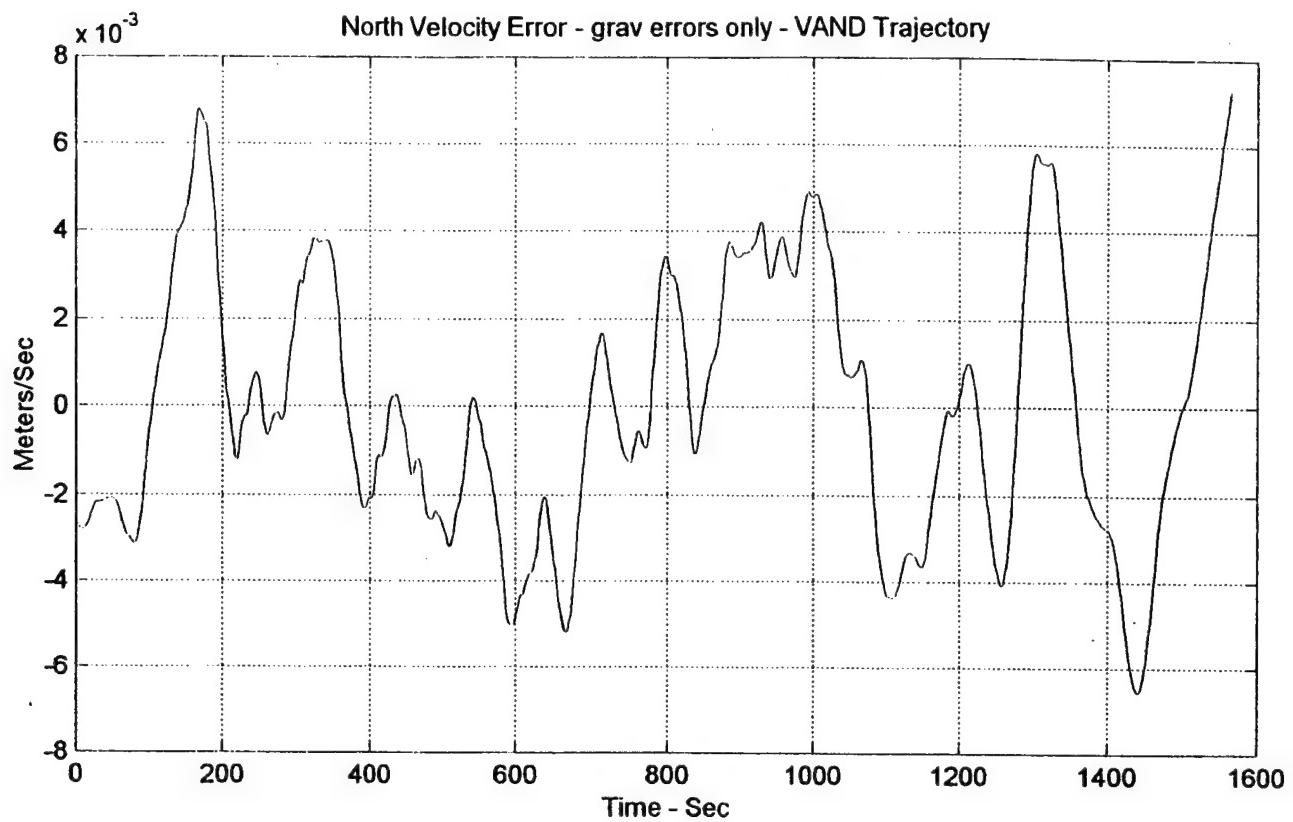
North Position Error - grav errors only - FK Trajectory



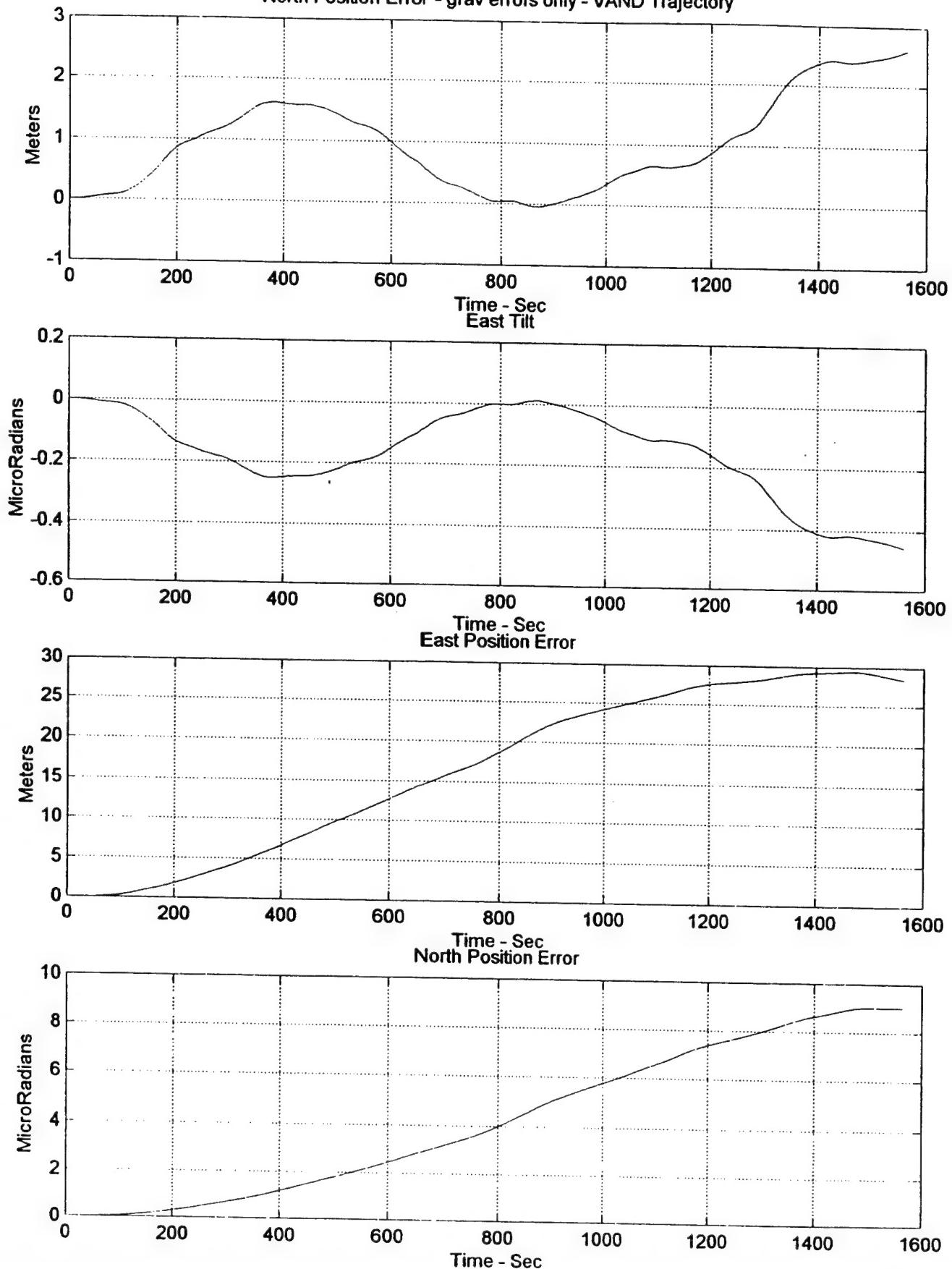


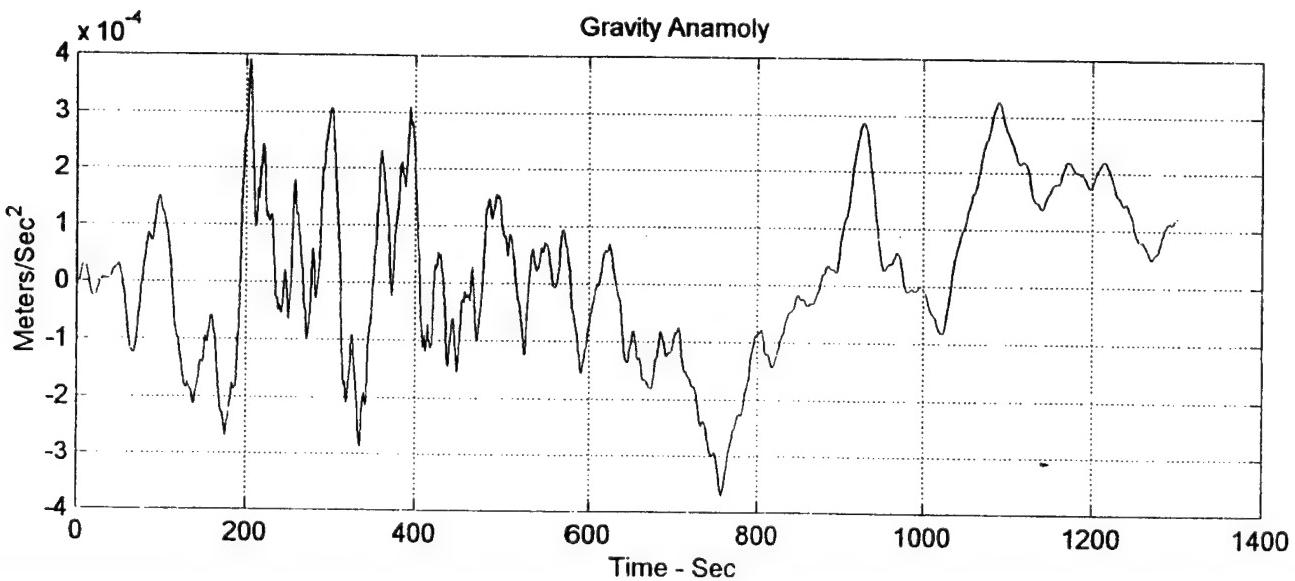
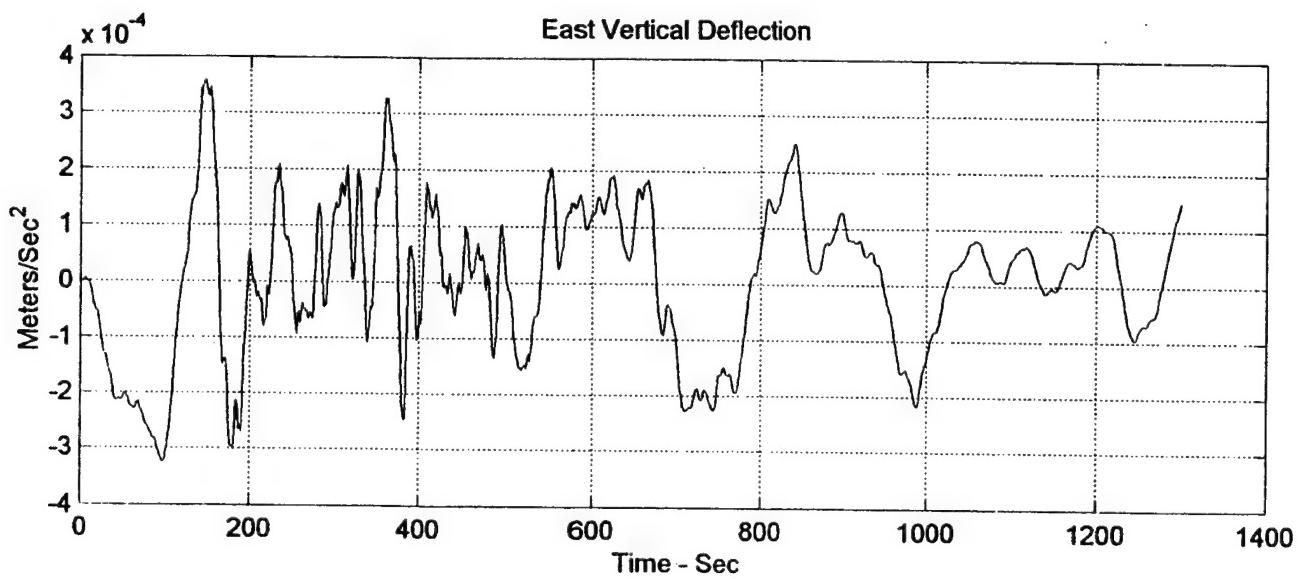
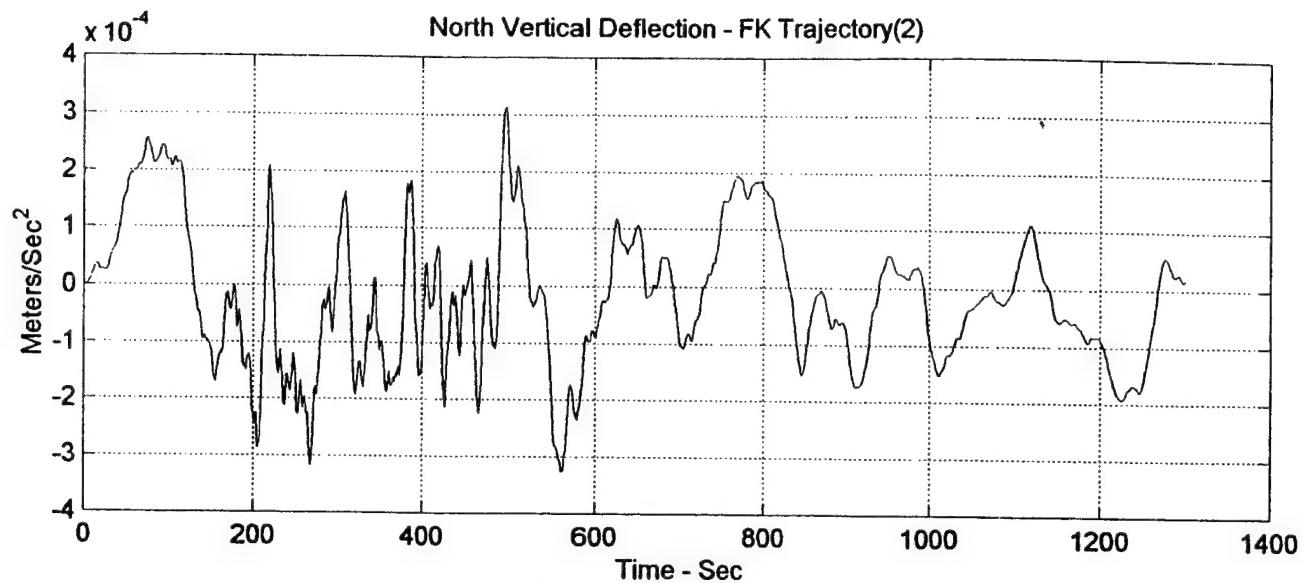
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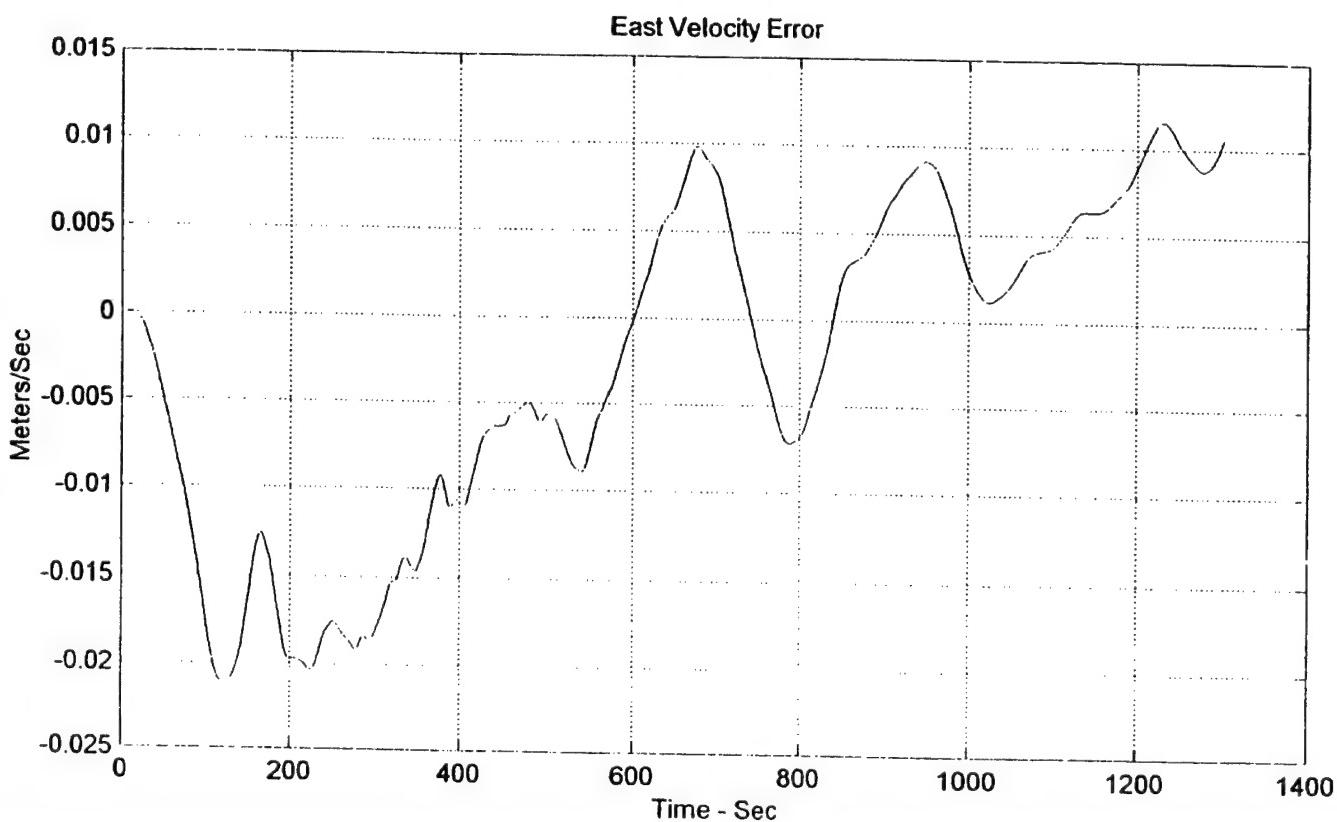
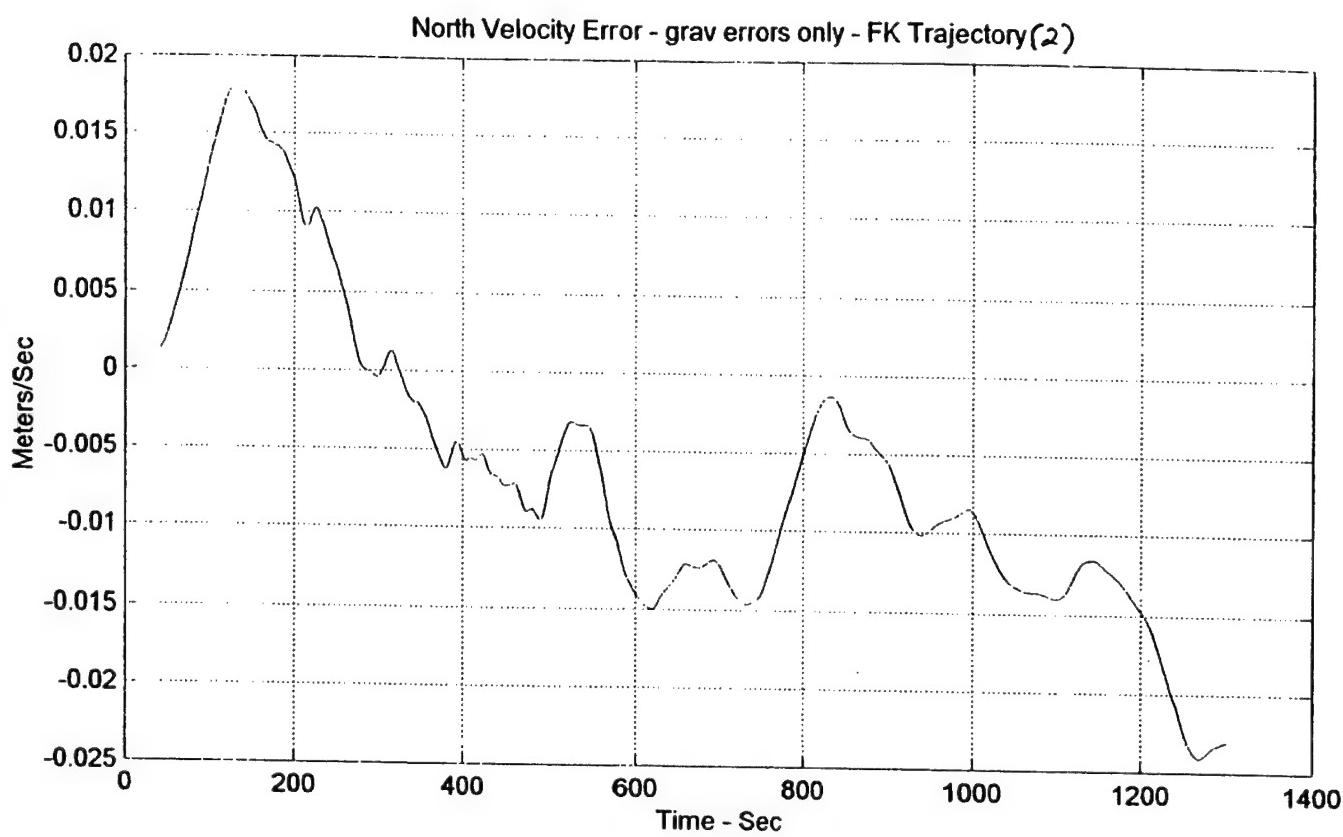


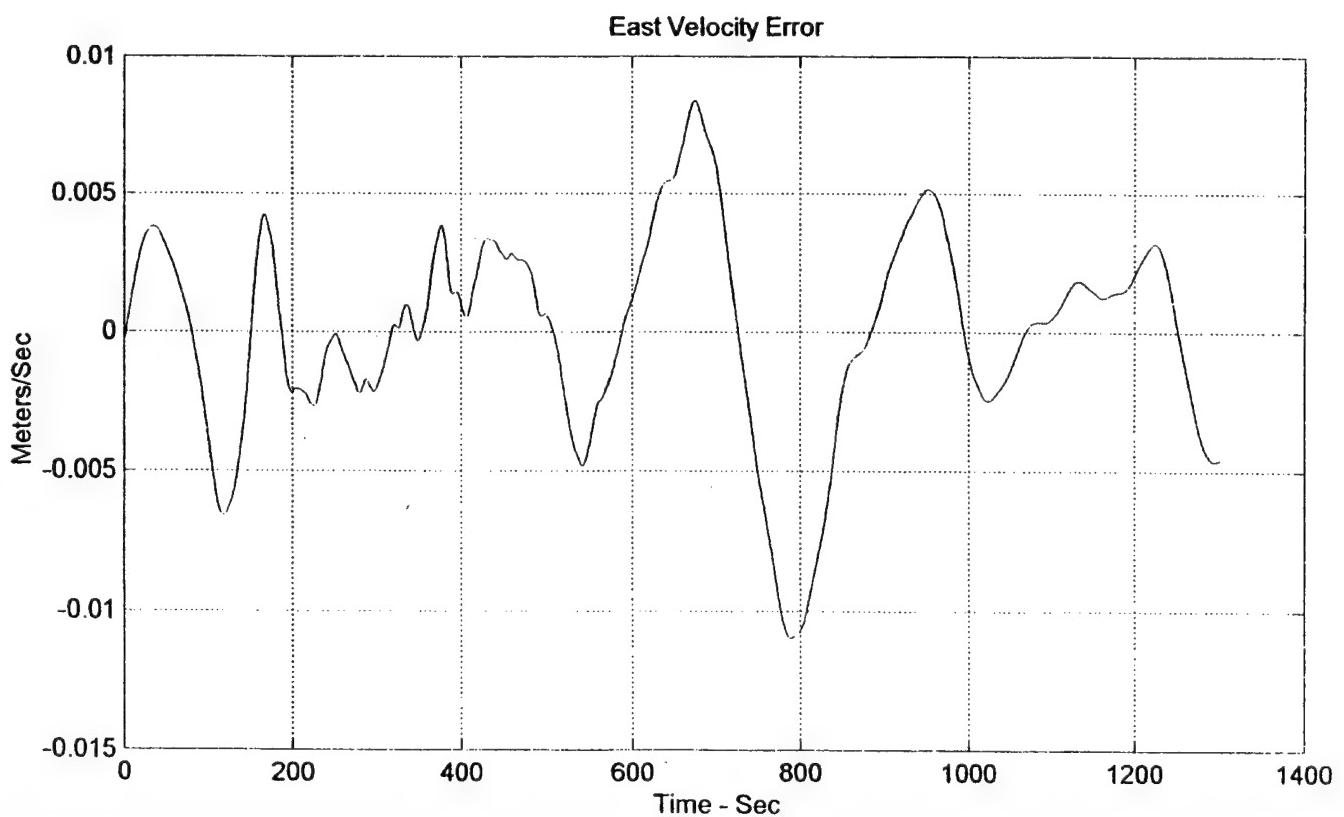
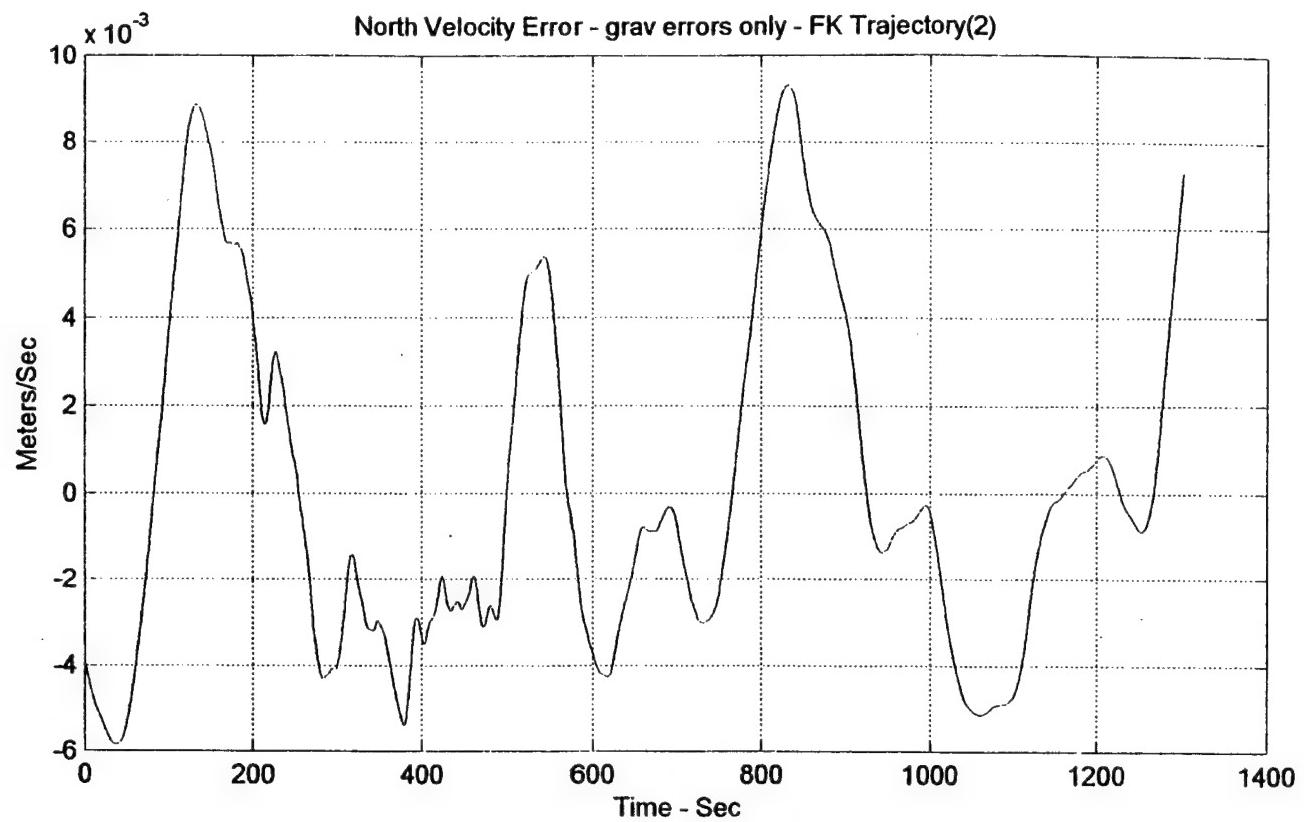


North Position Error - grav errors only - VAND Trajectory

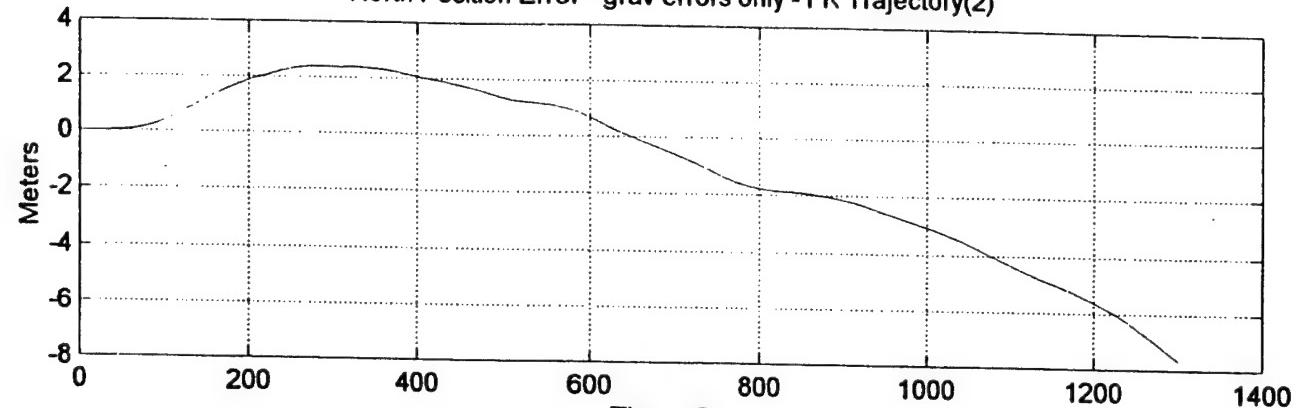




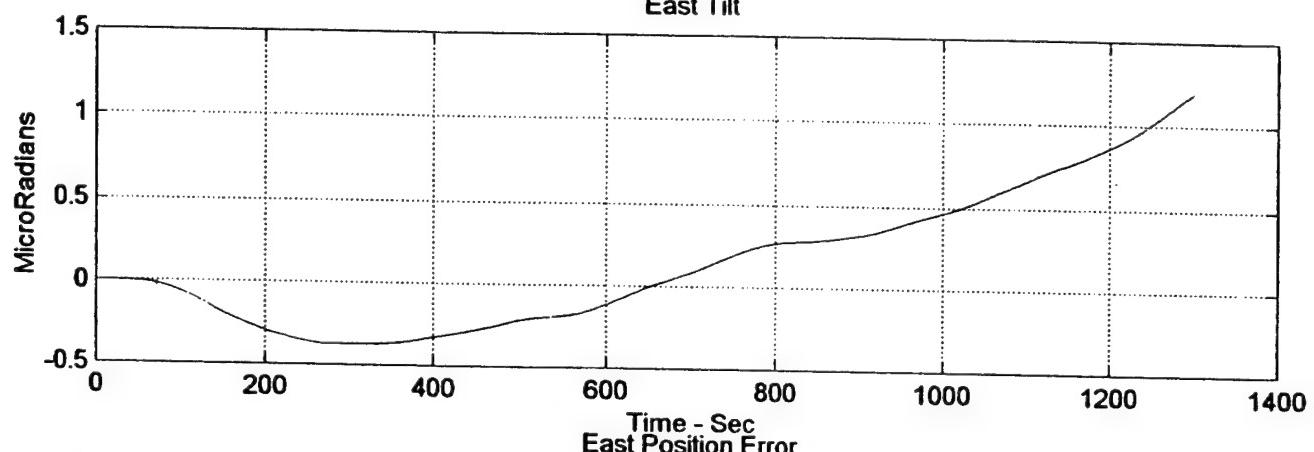




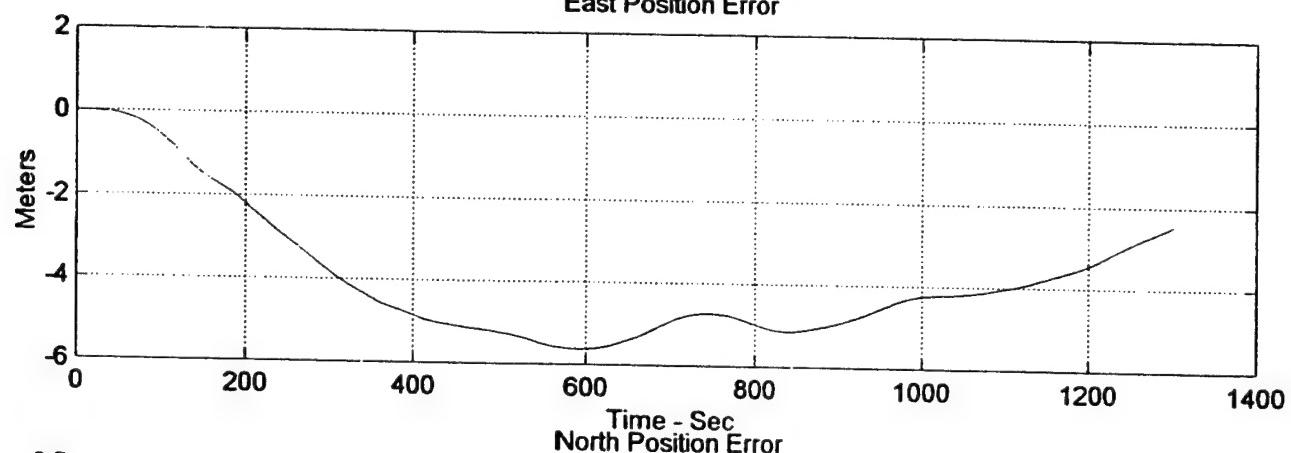
North Position Error - grav errors only - FK Trajectory(2)



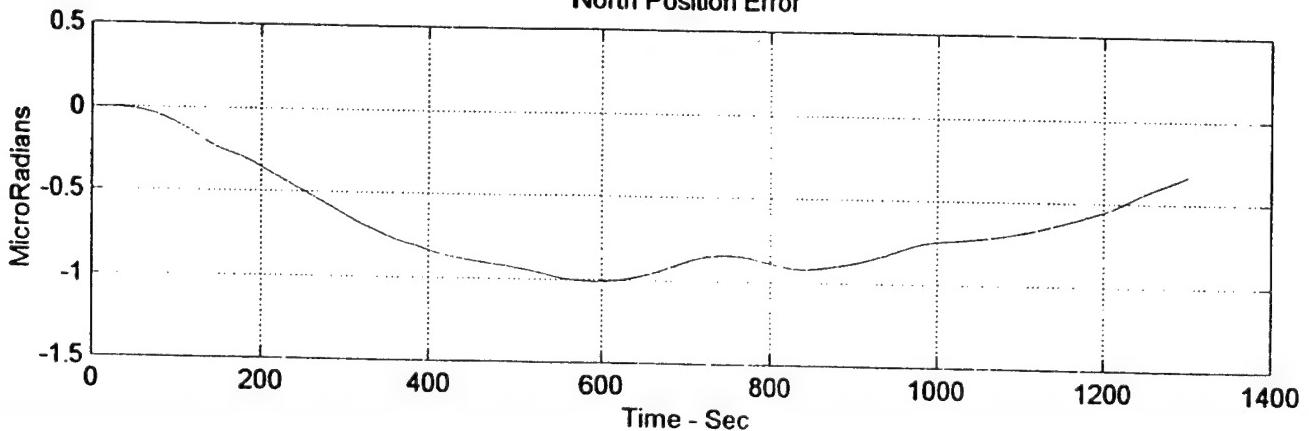
Time - Sec
East Tilt



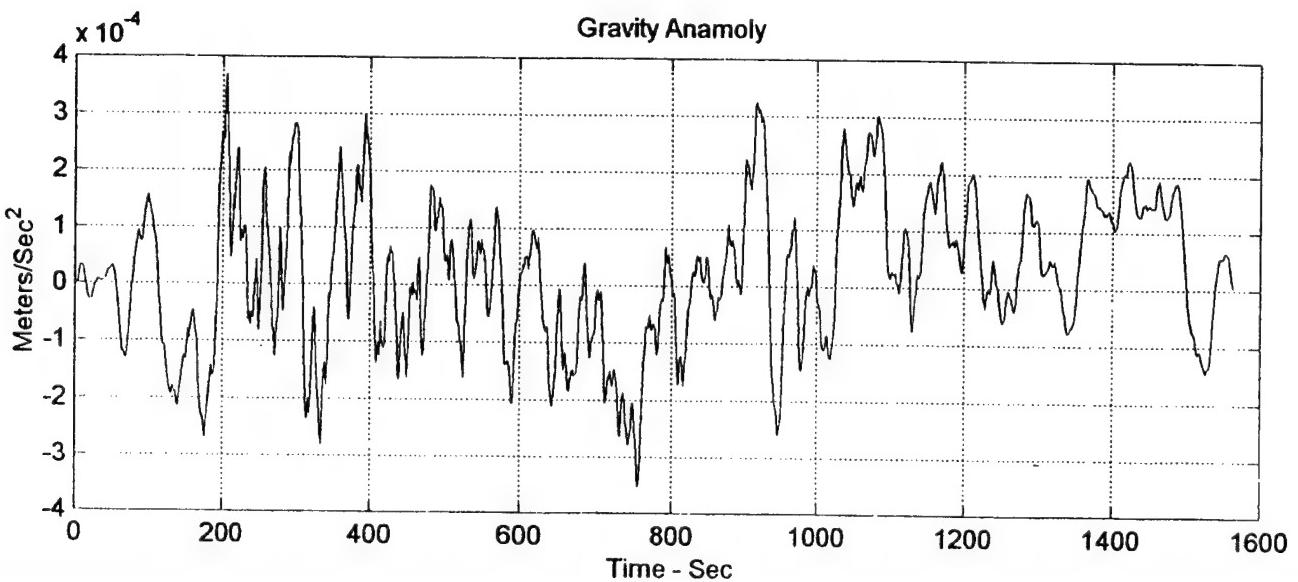
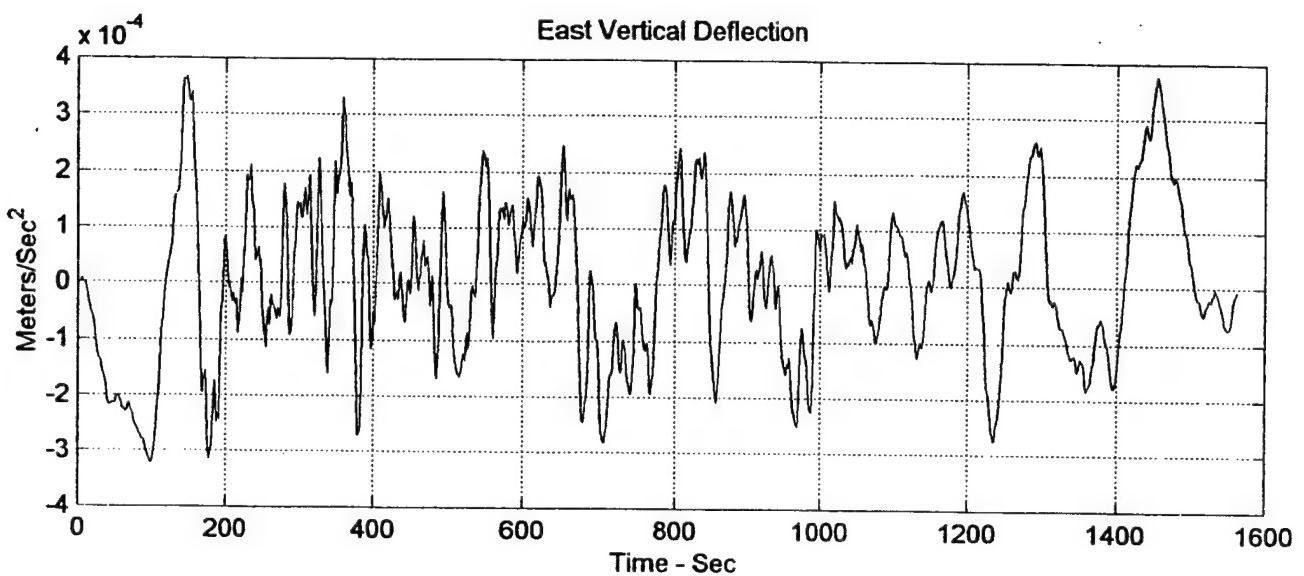
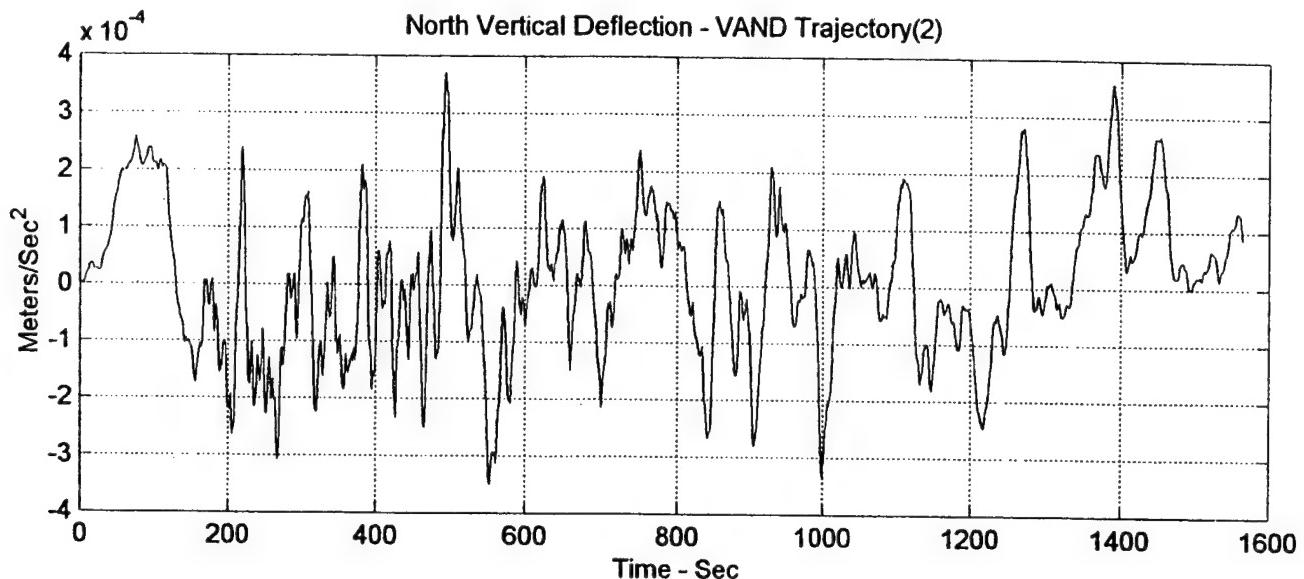
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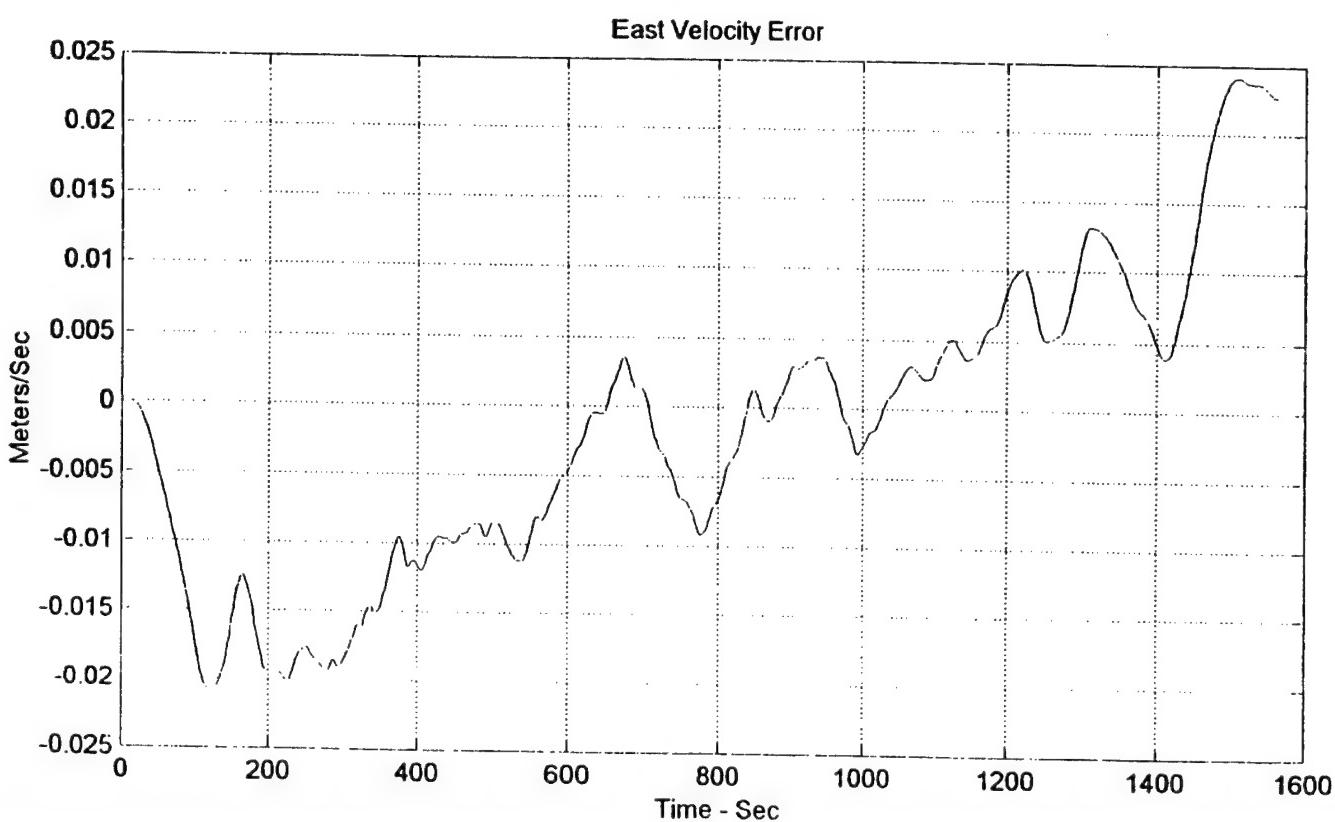
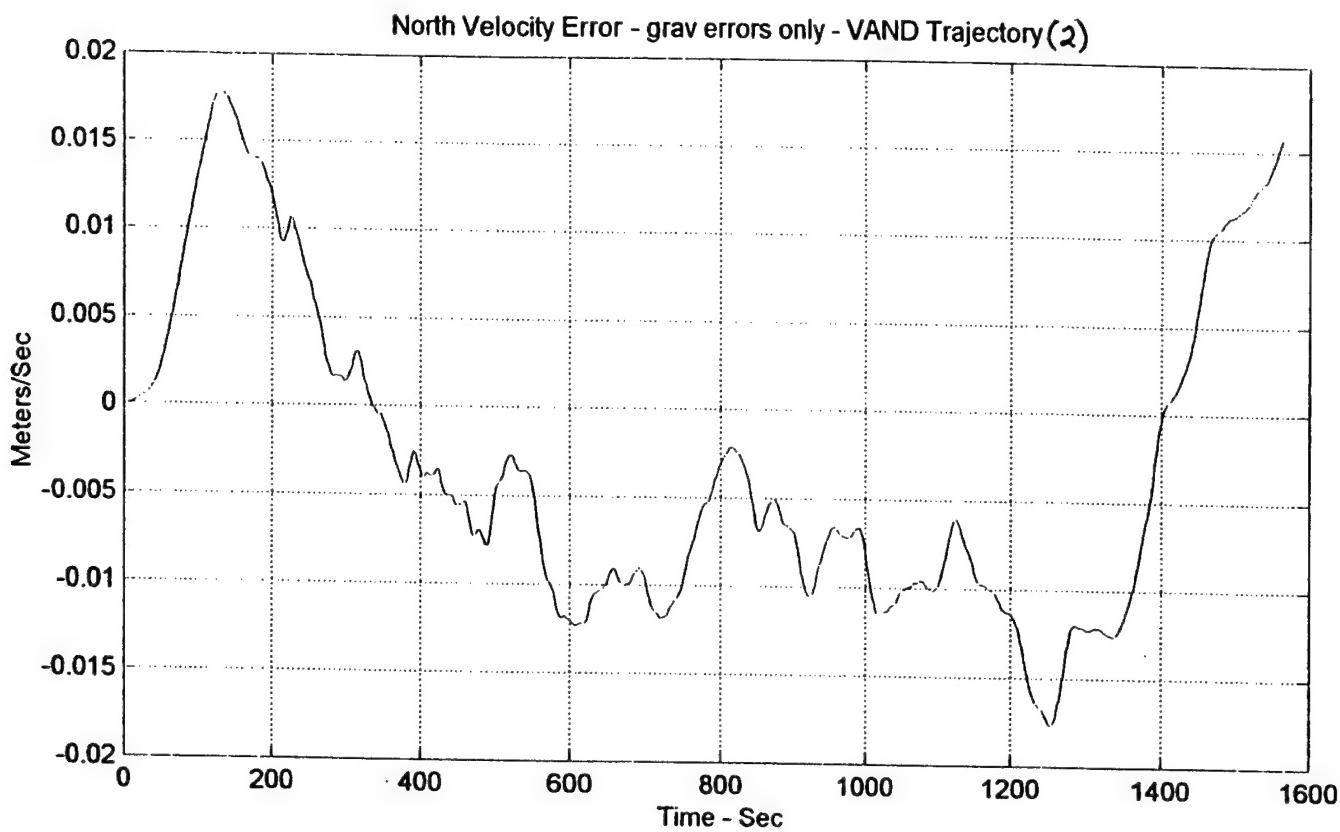


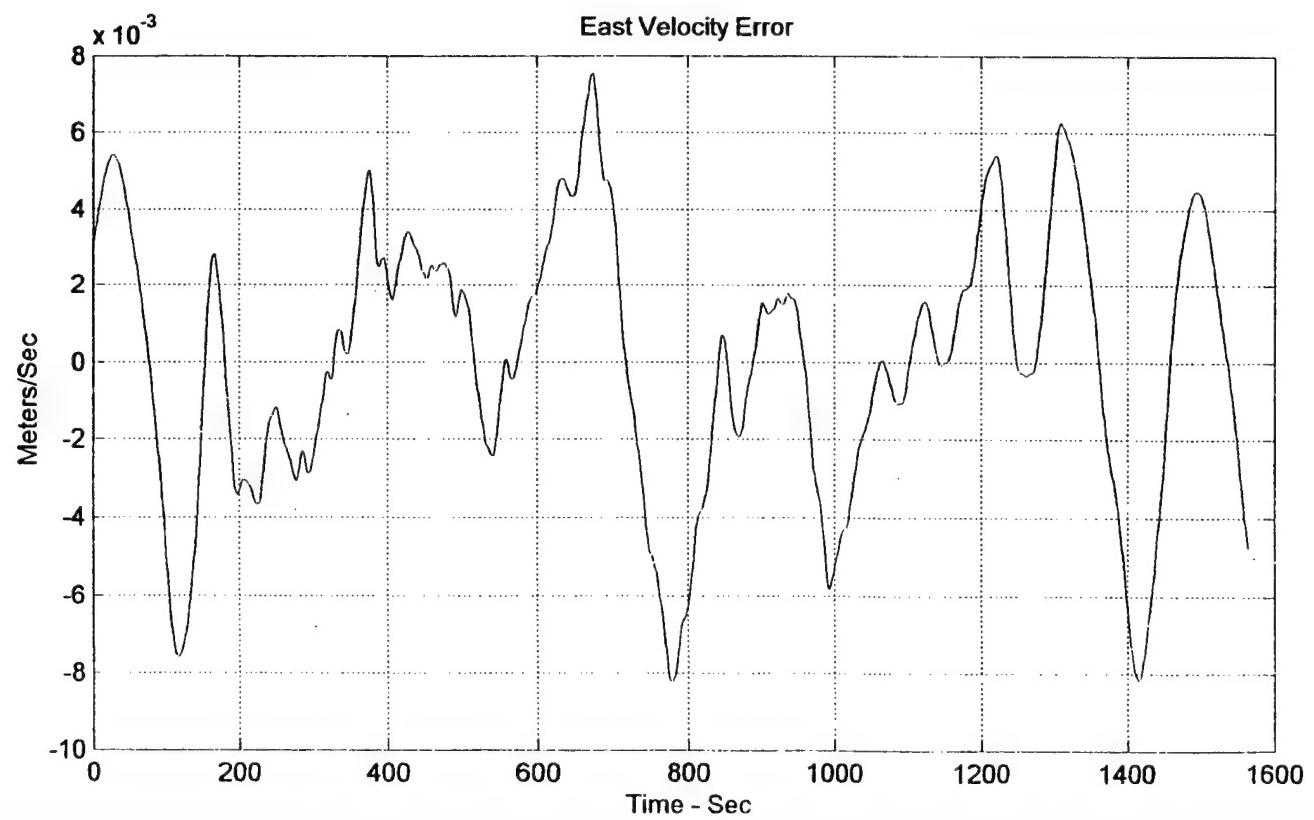
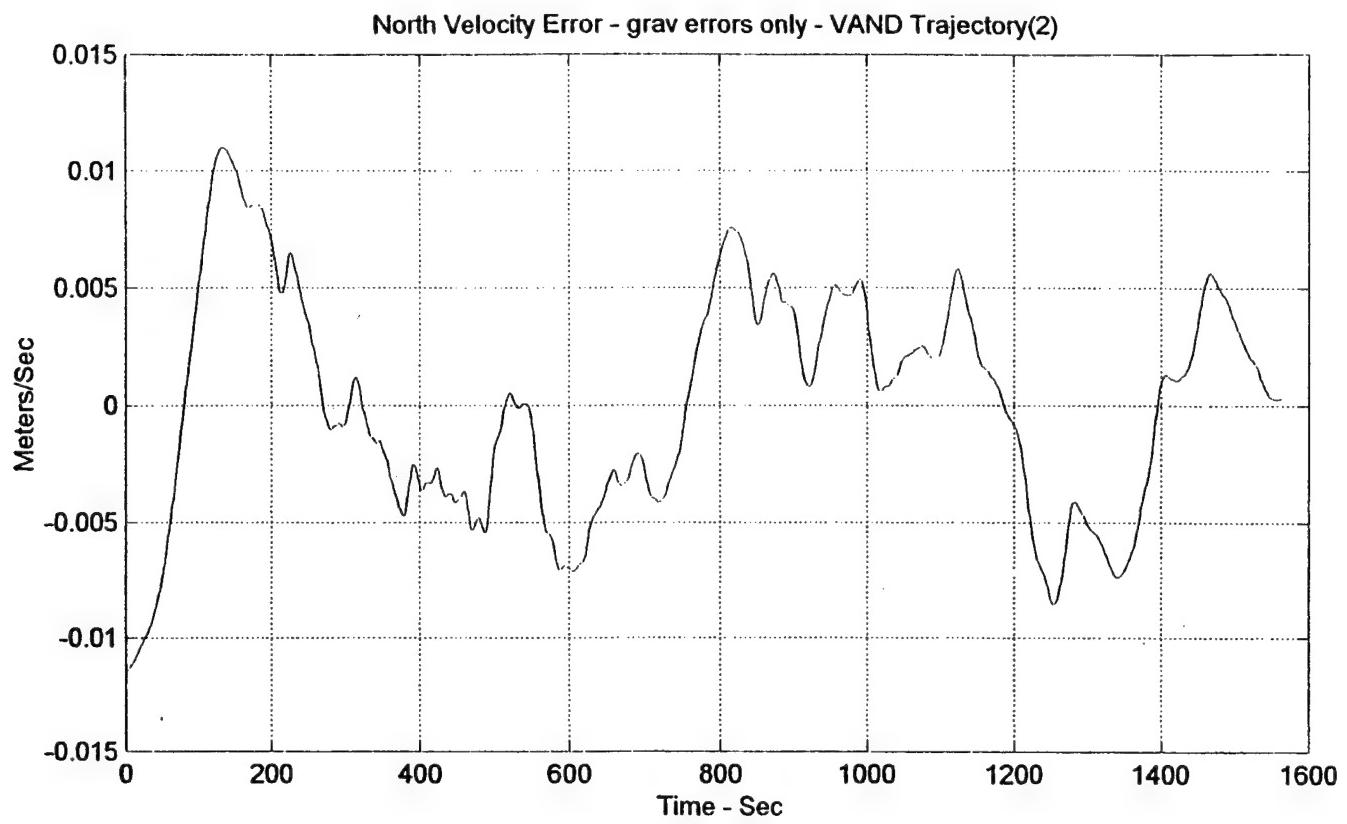
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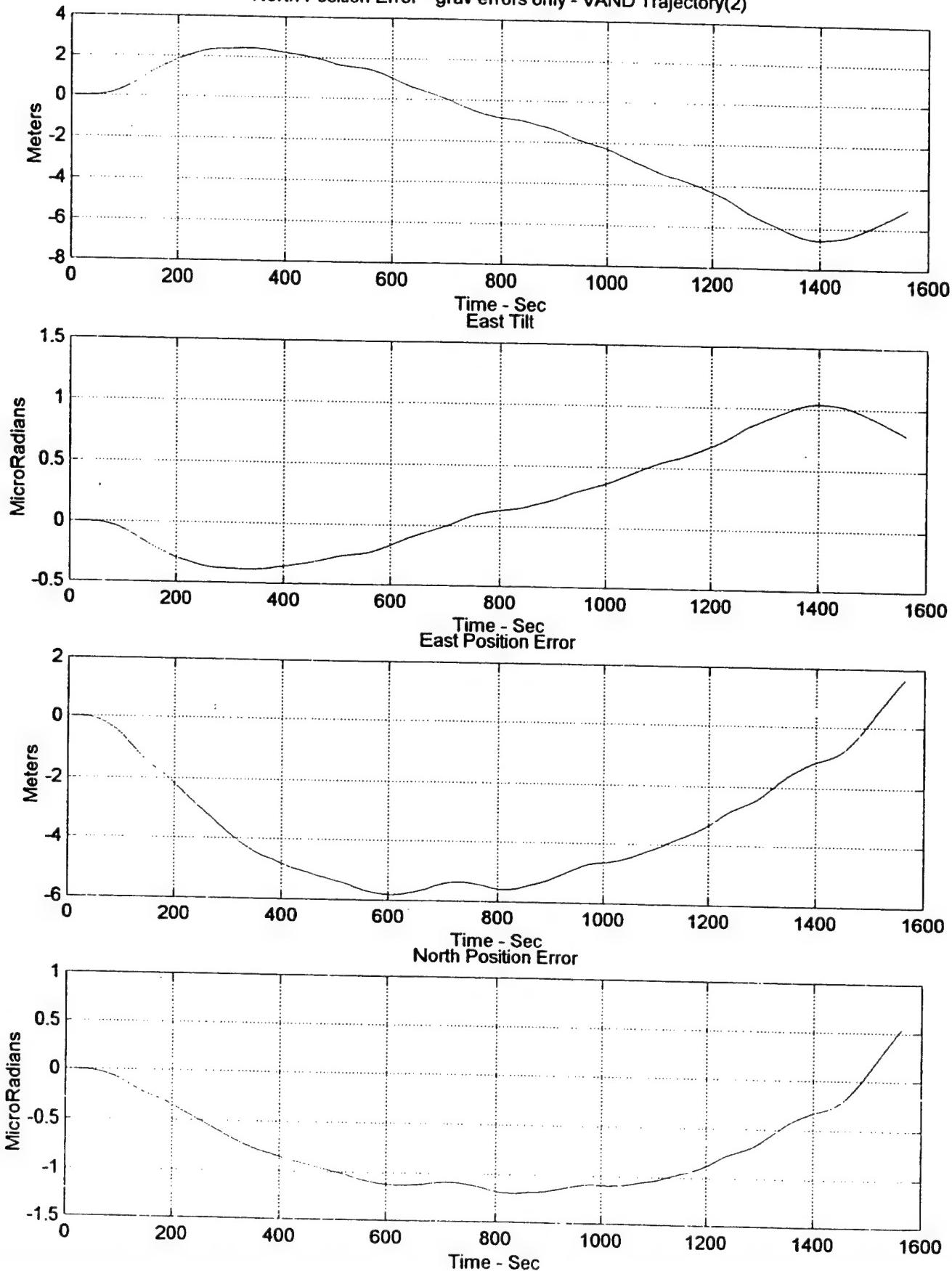
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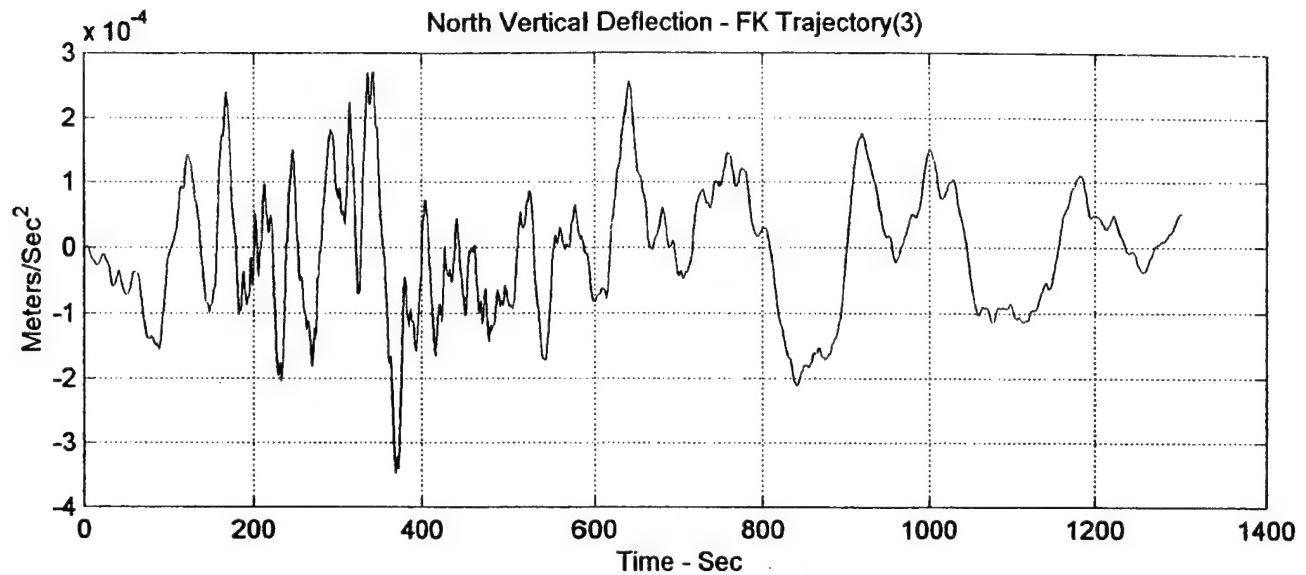




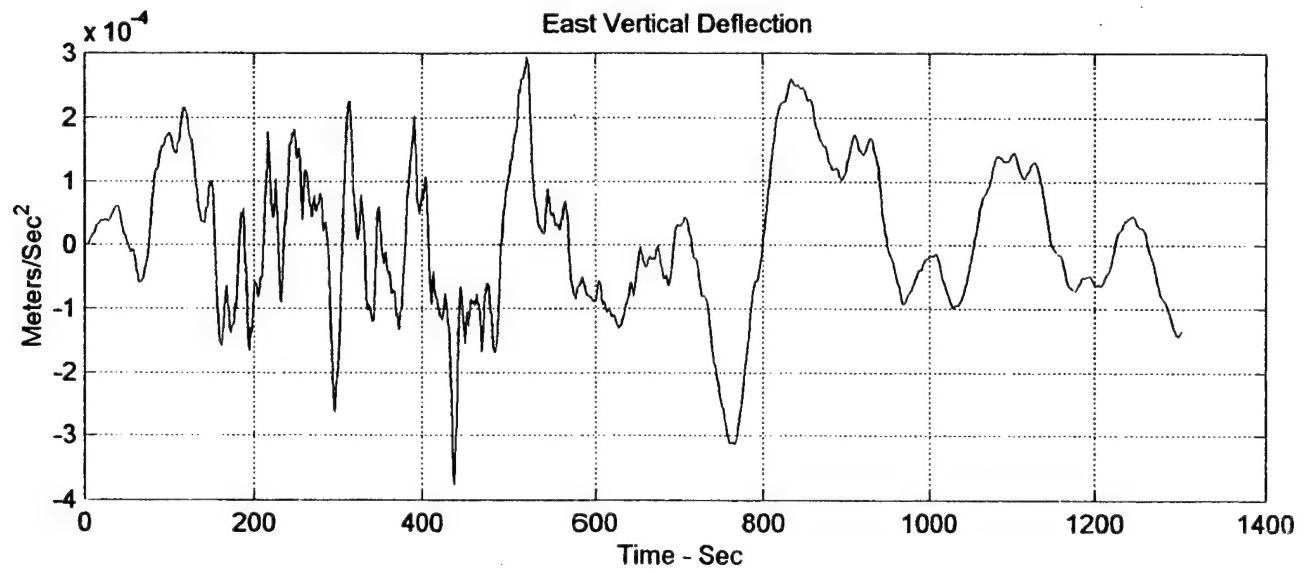
North Position Error - grav errors only - VAND Trajectory(2)



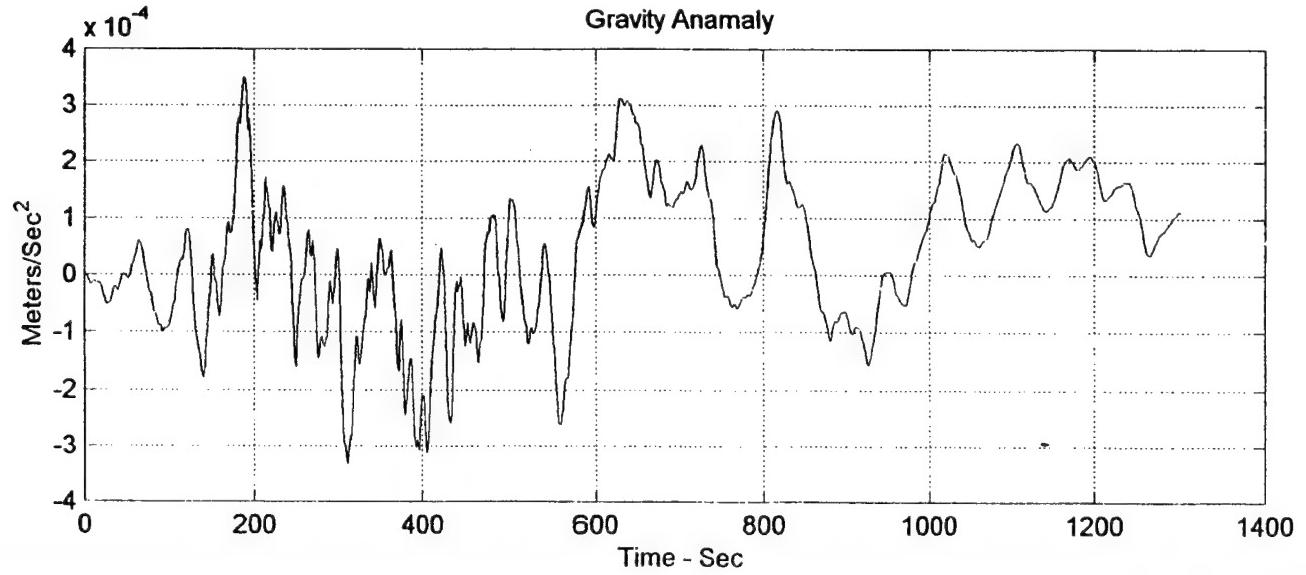
North Vertical Deflection - FK Trajectory(3)

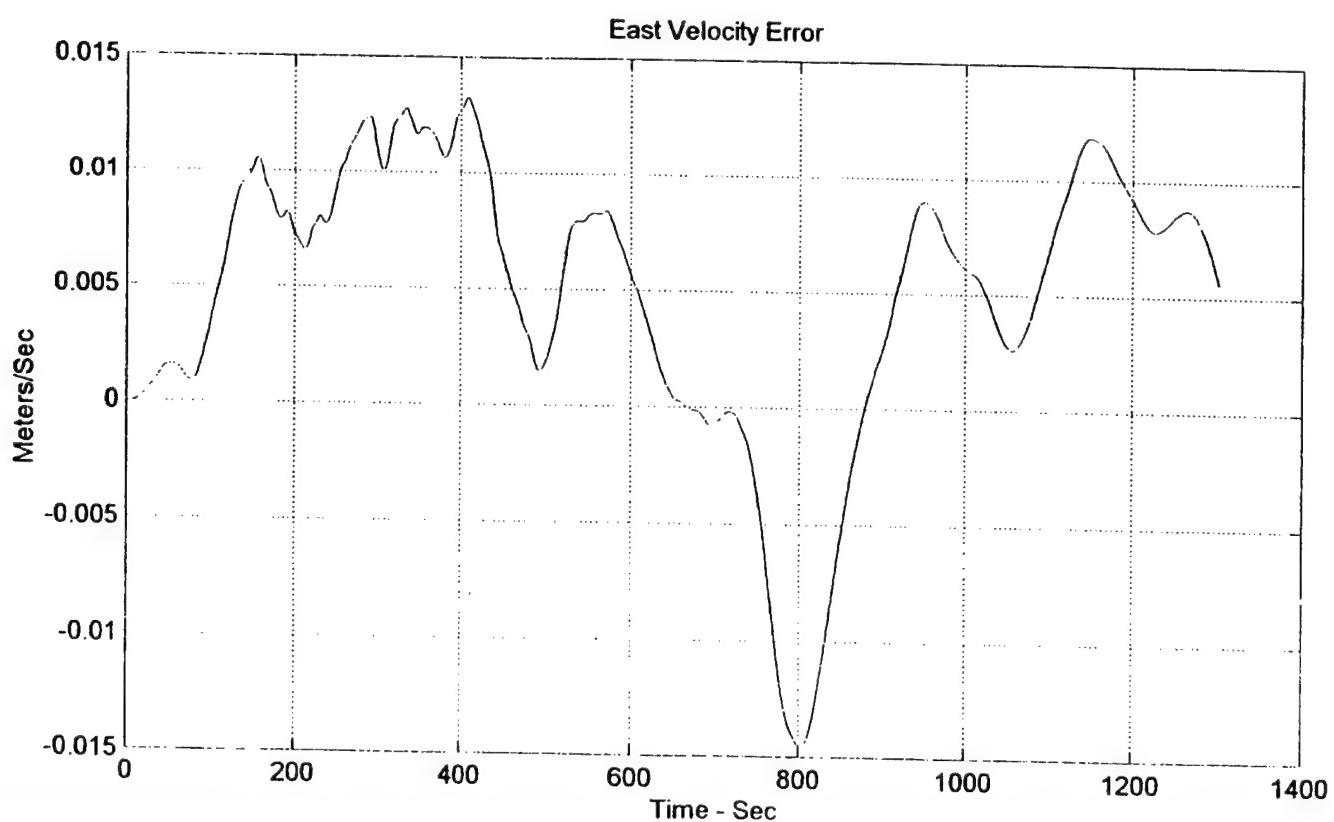
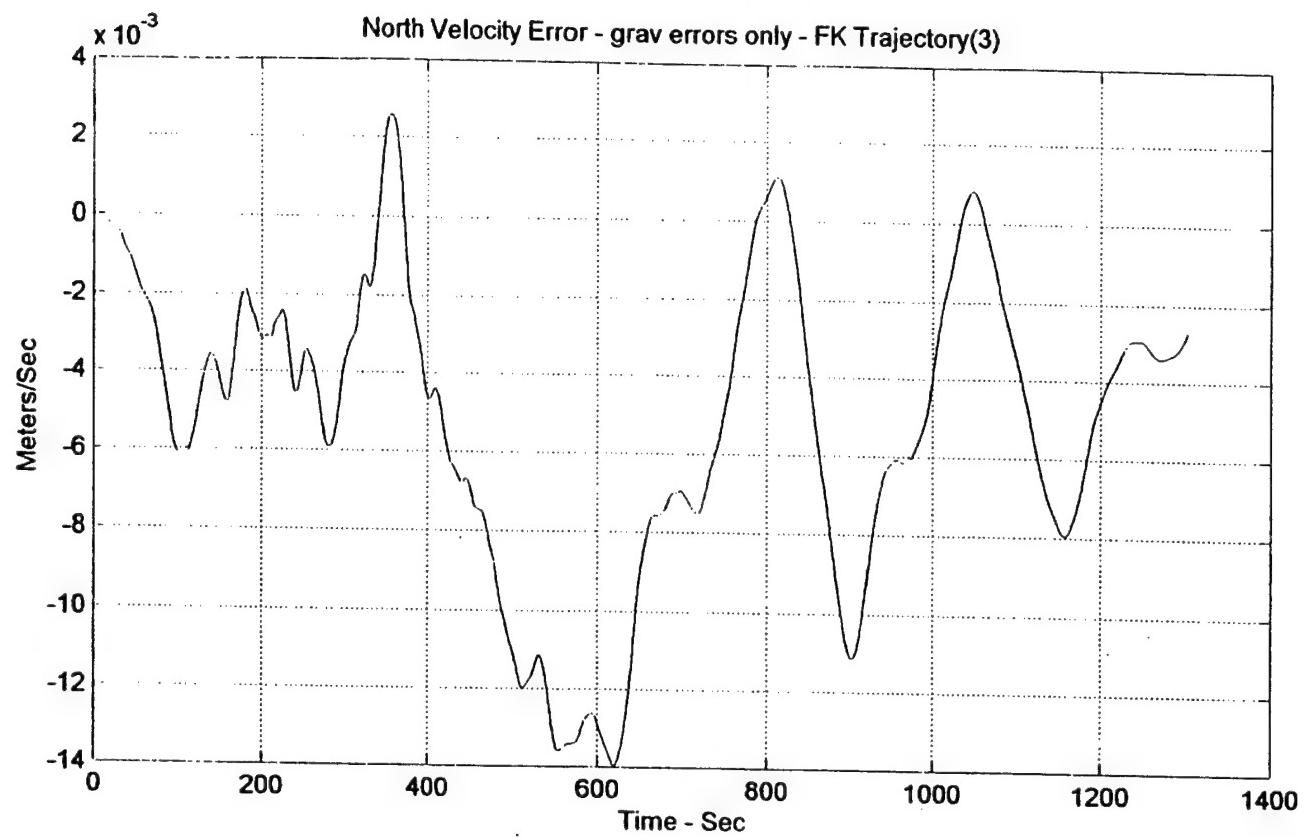


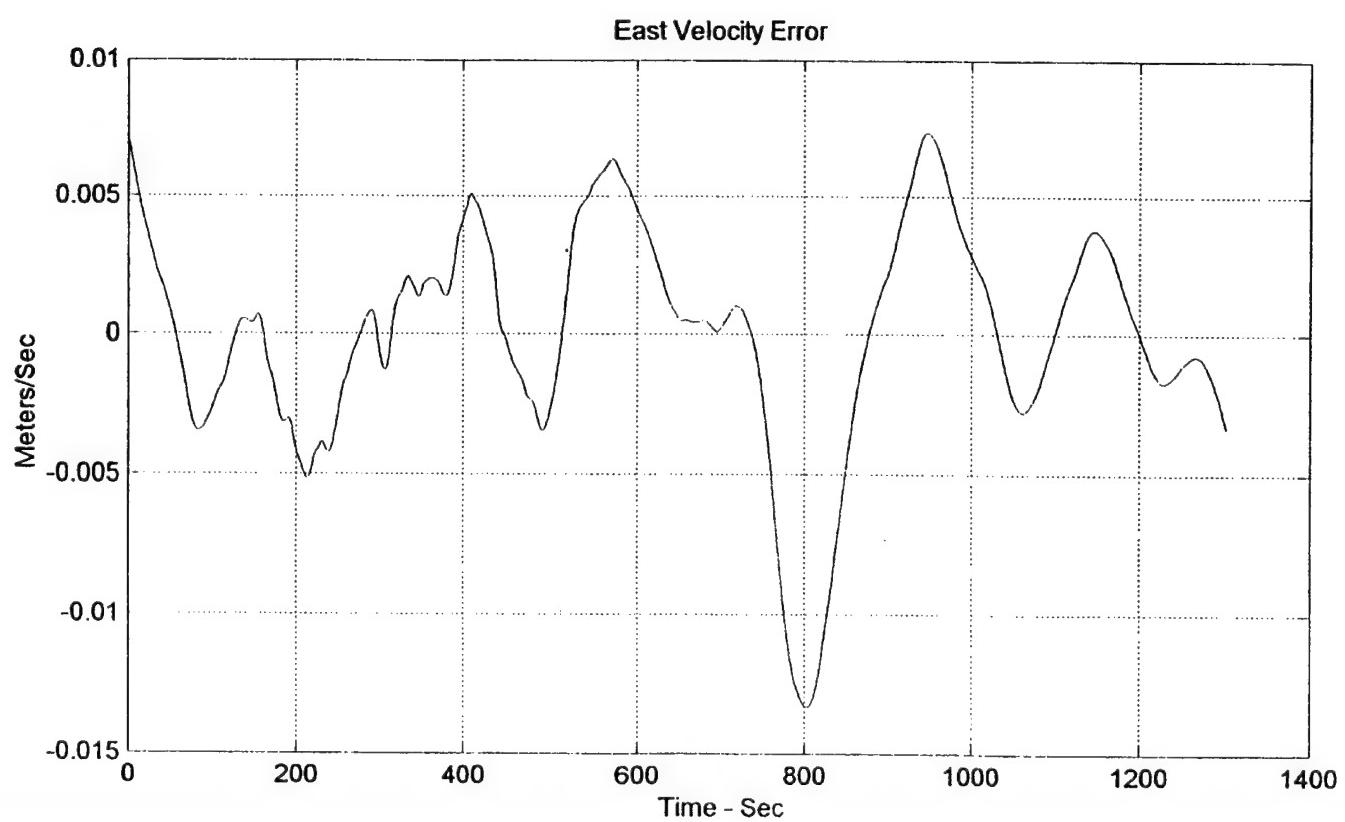
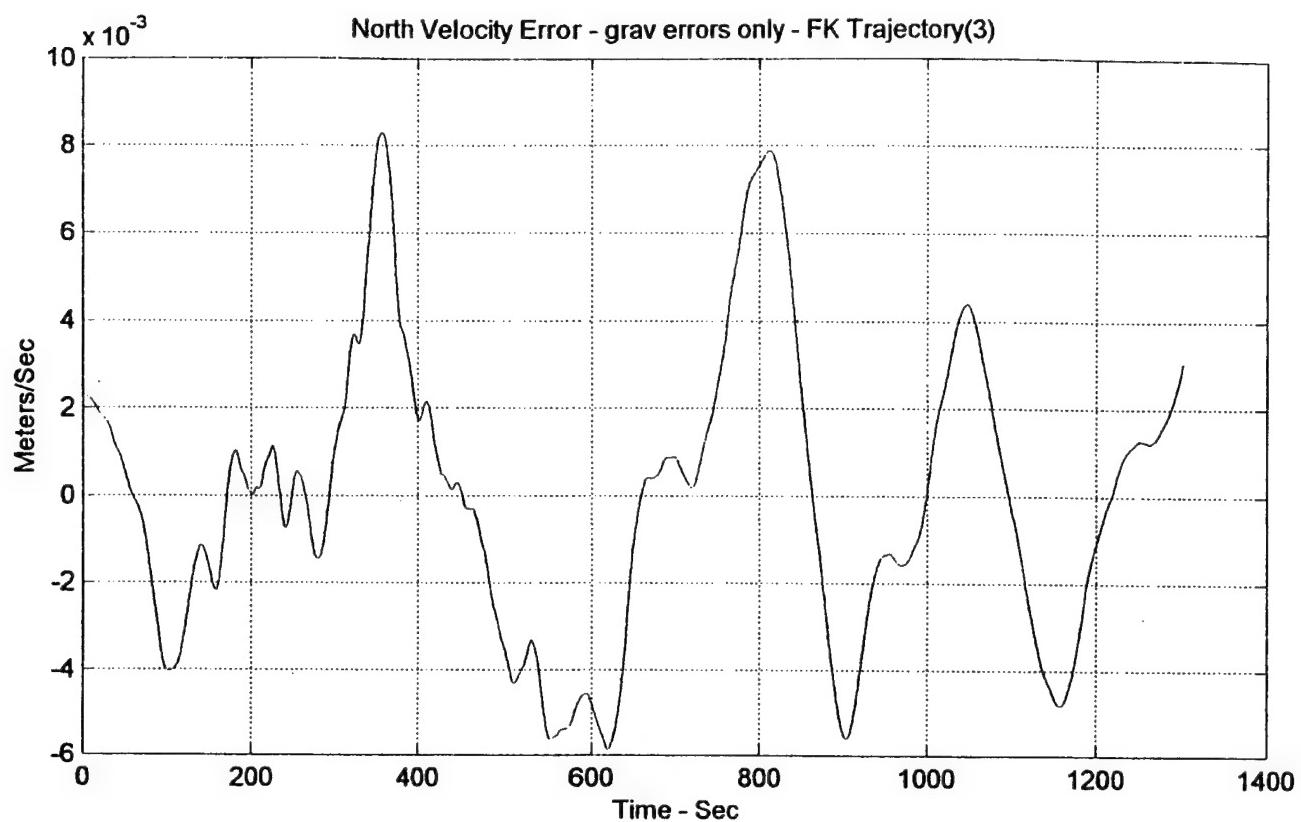
East Vertical Deflection



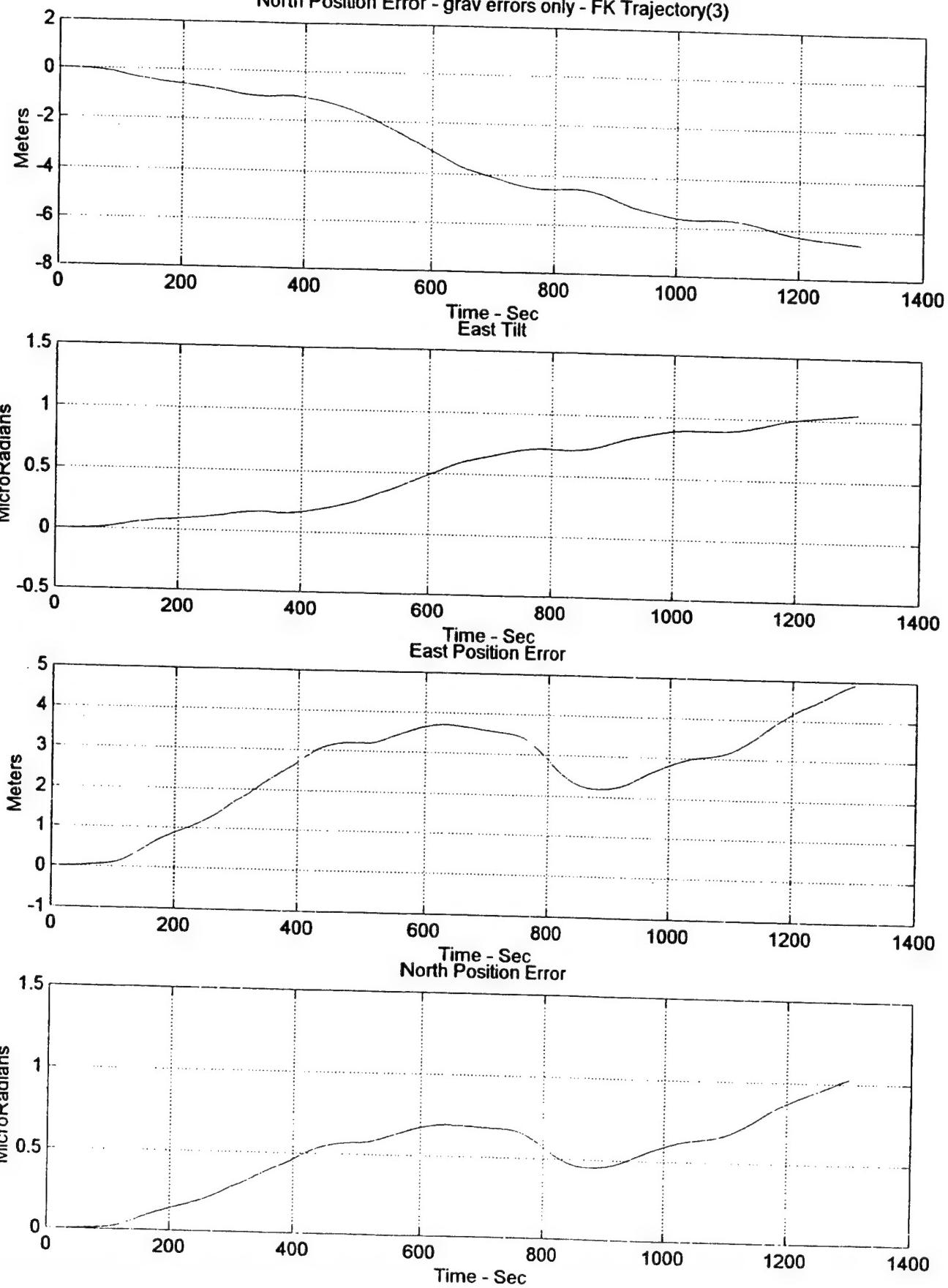
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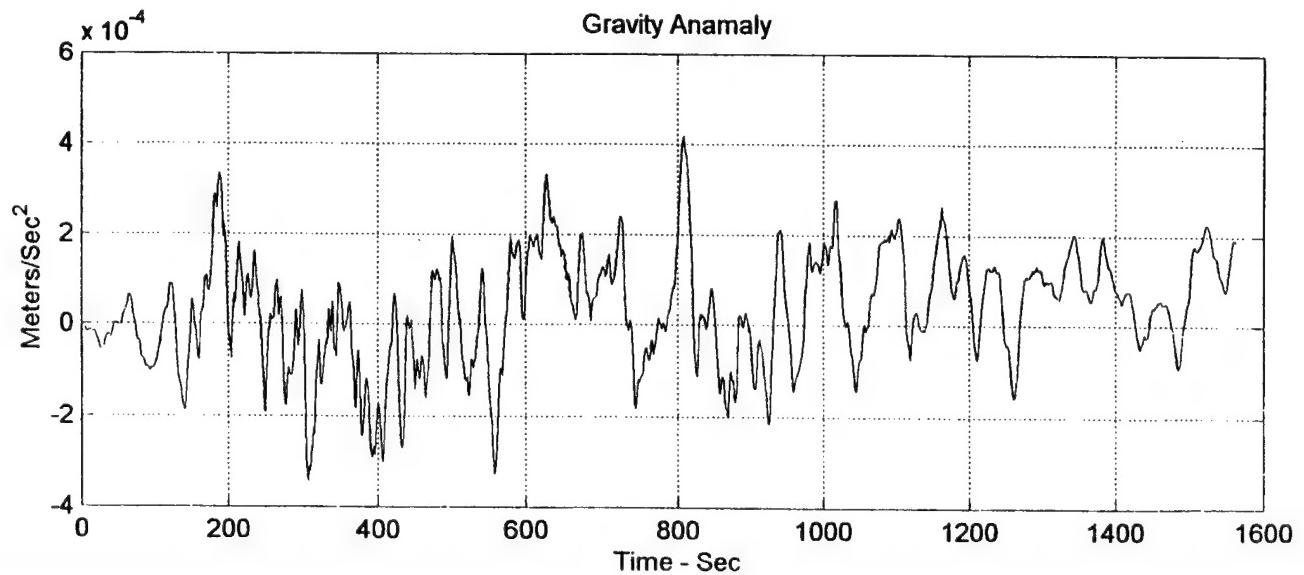
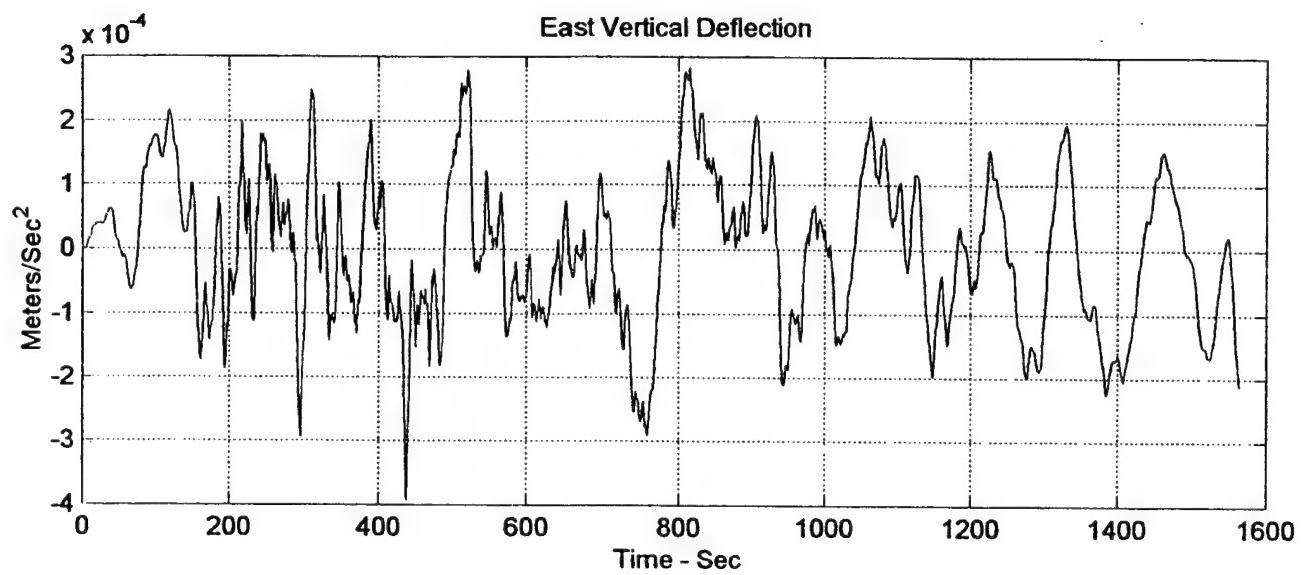
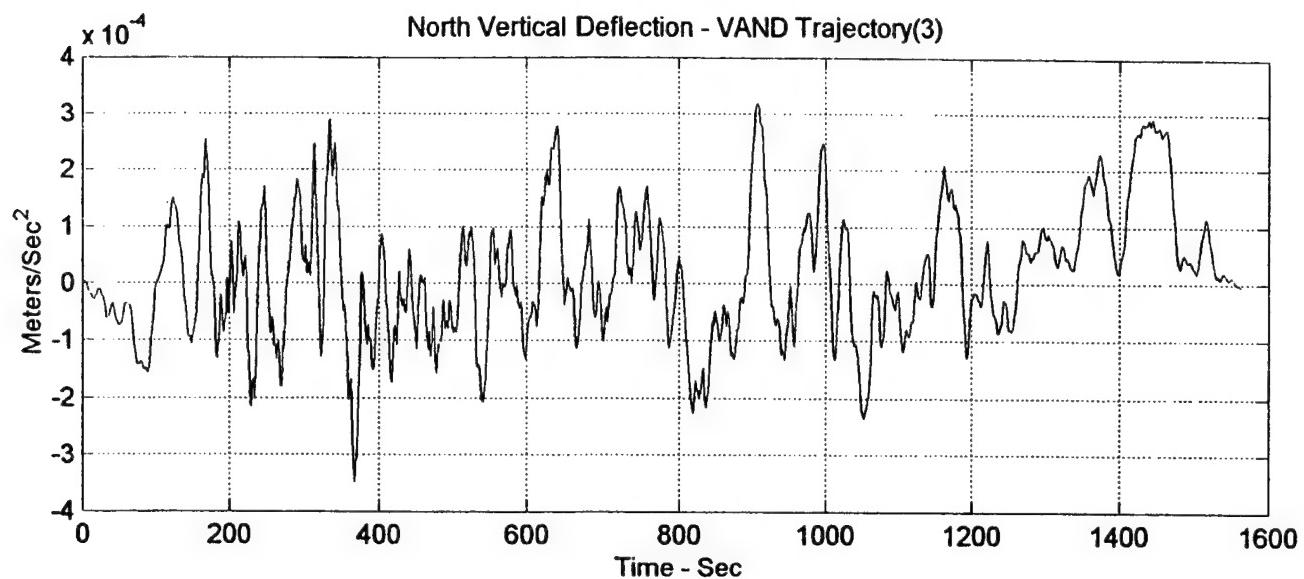


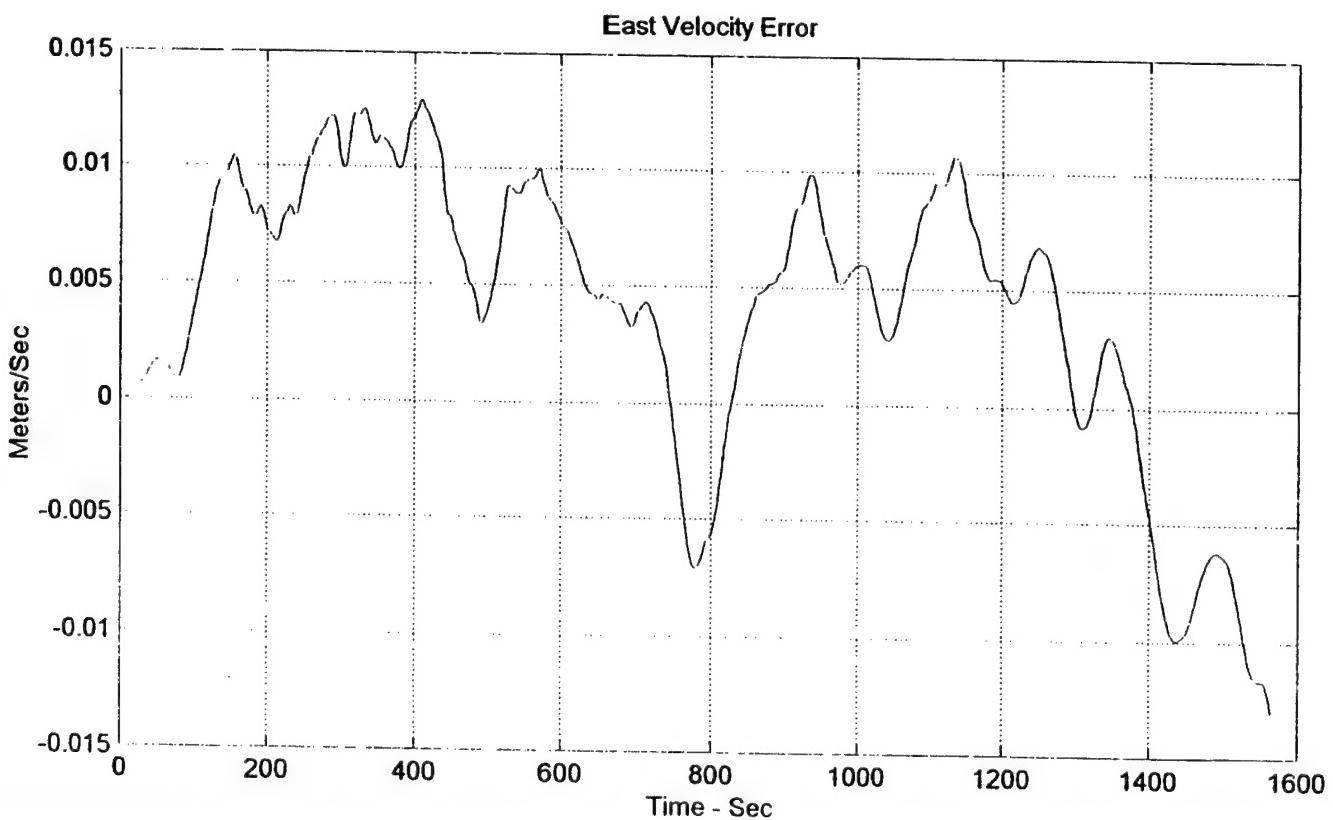
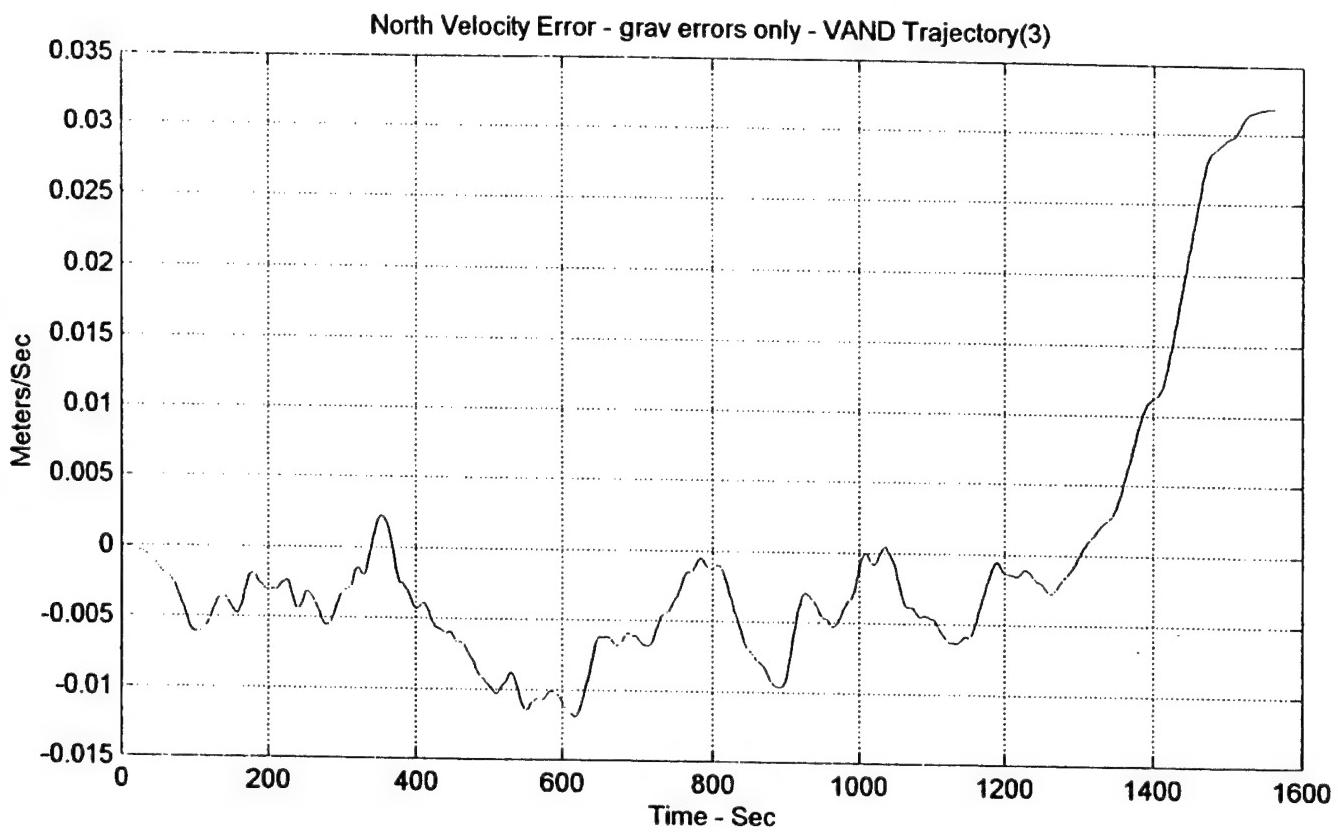


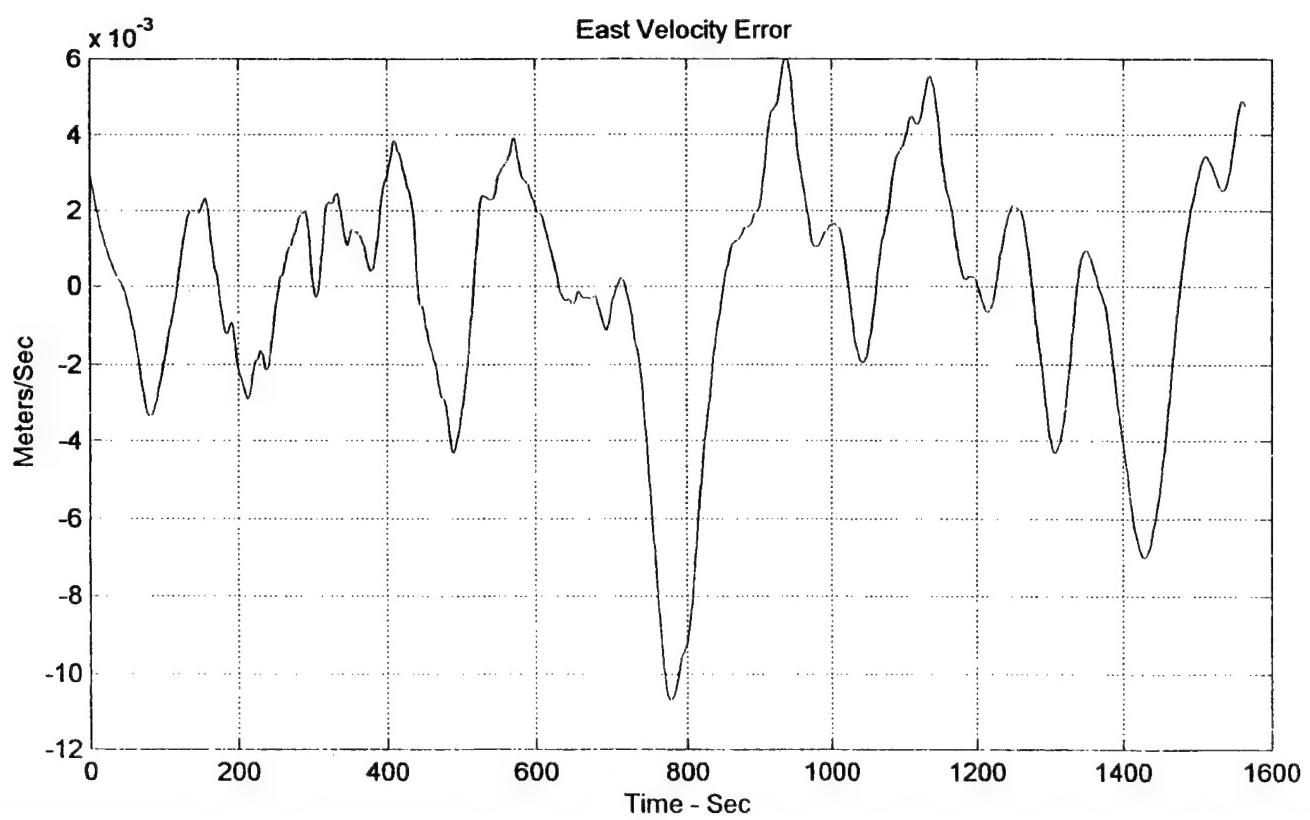
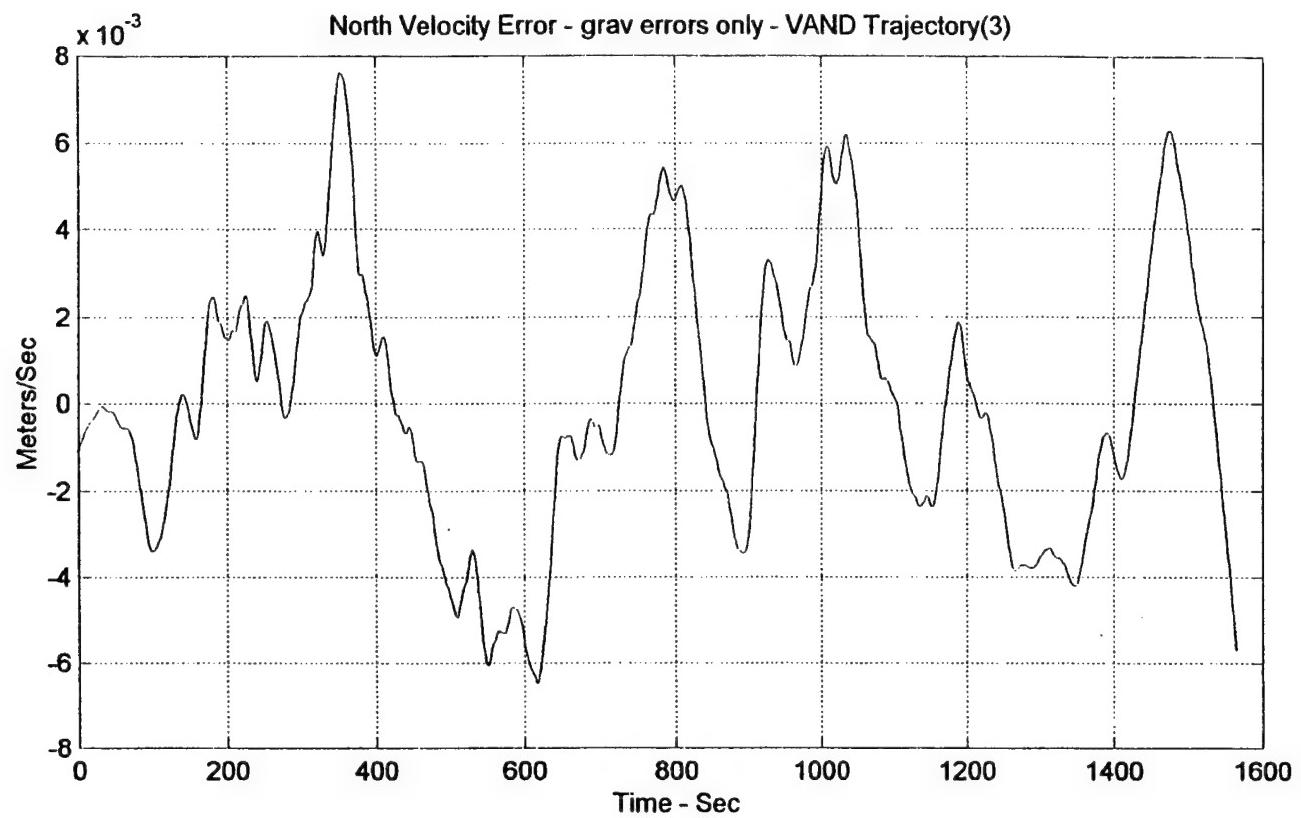


North Position Error - grav errors only - FK Trajectory(3)

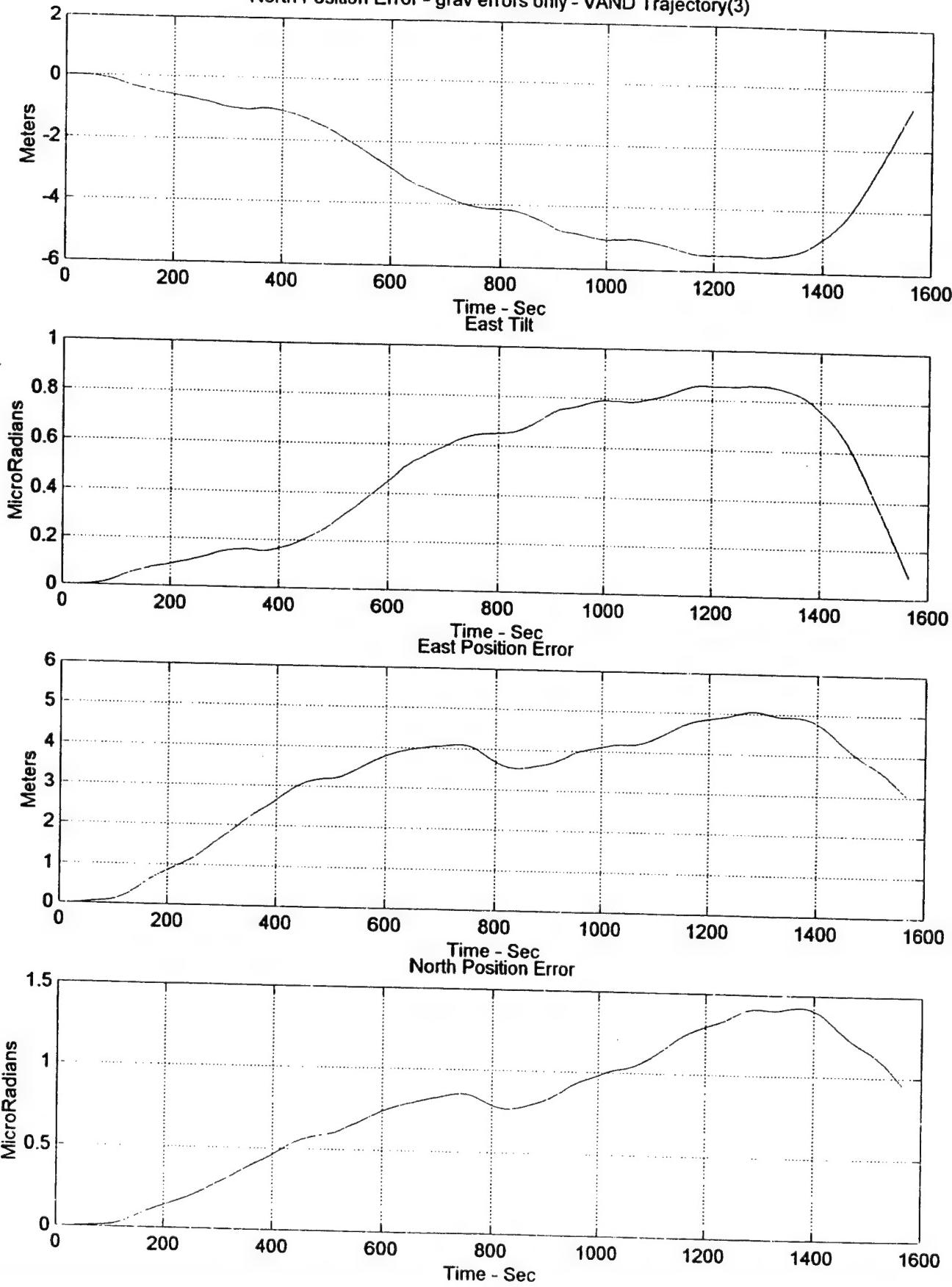




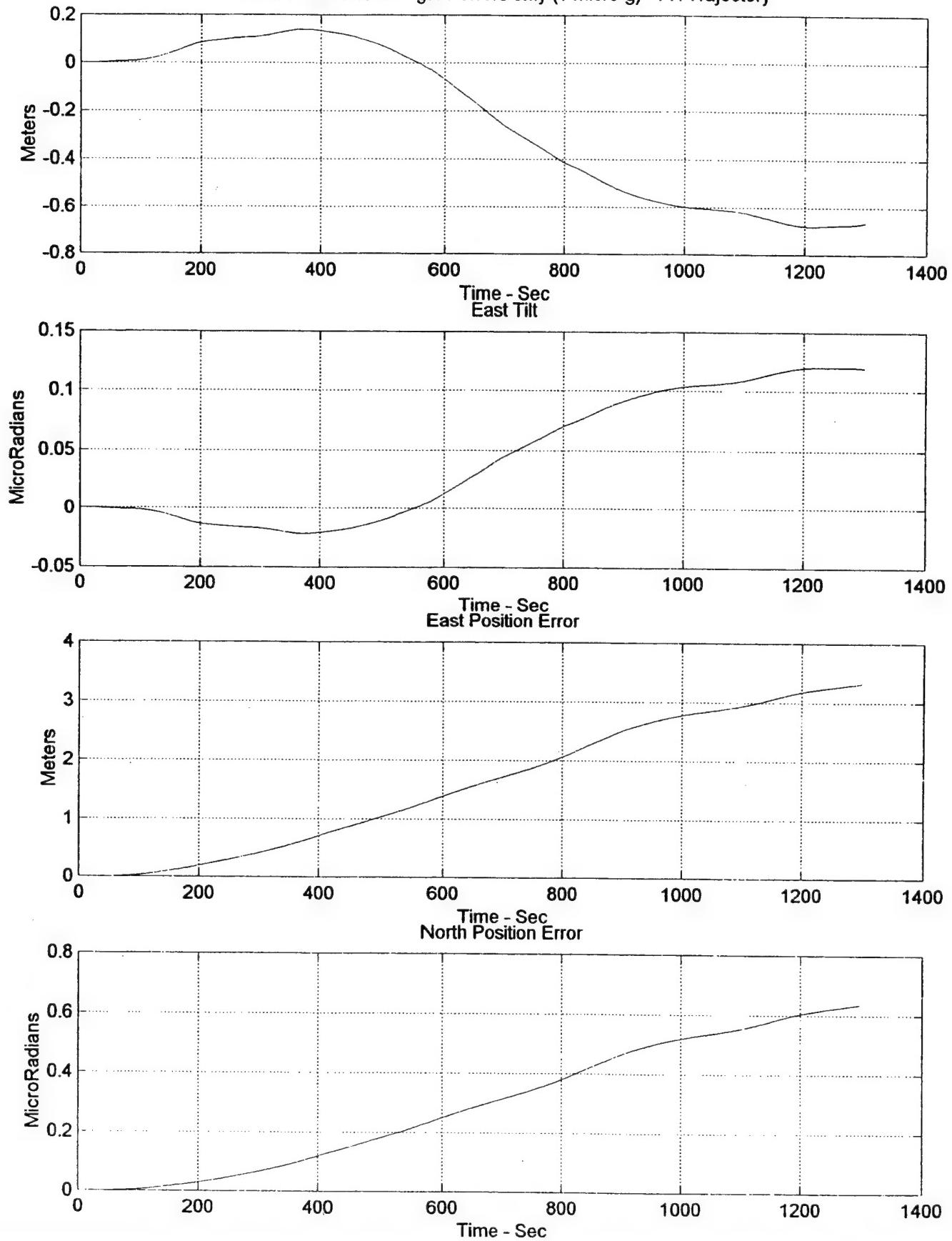


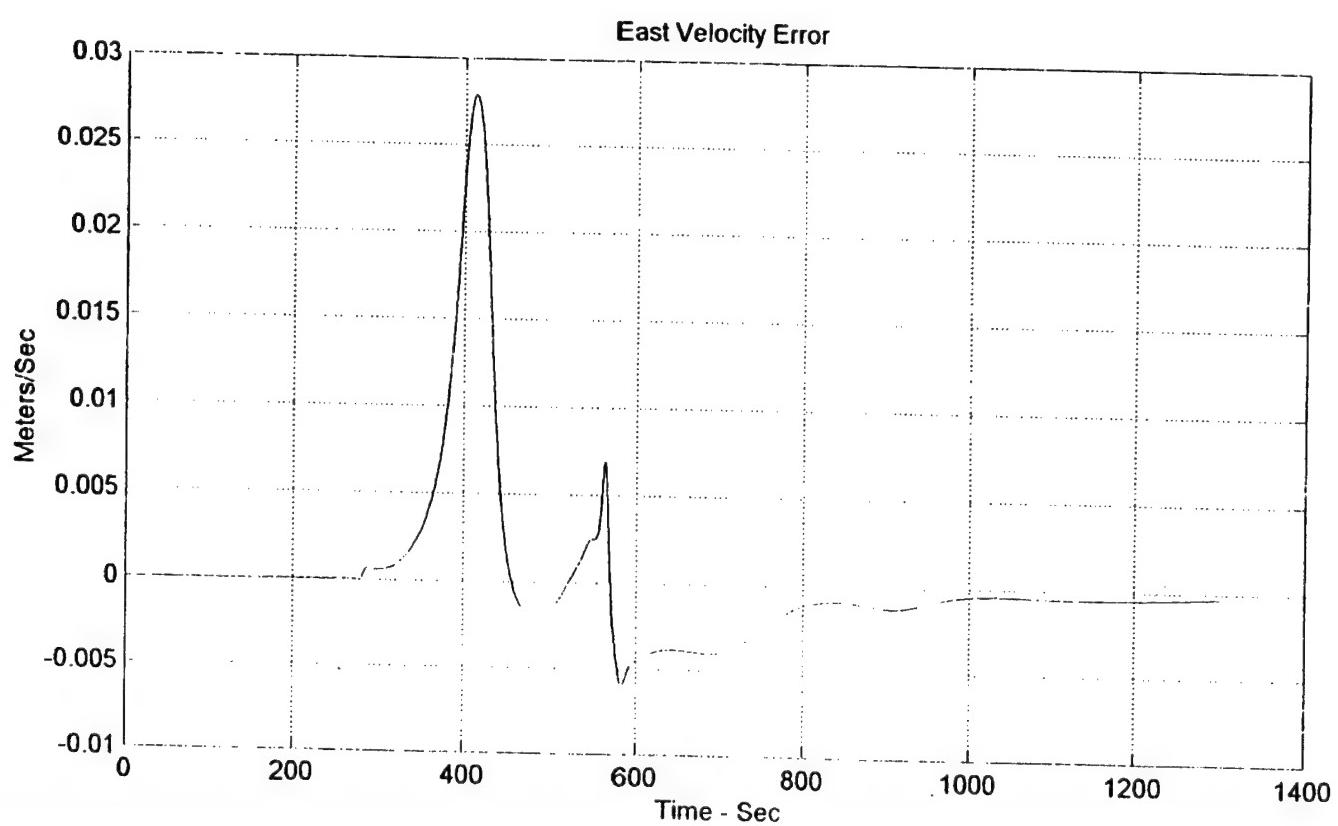
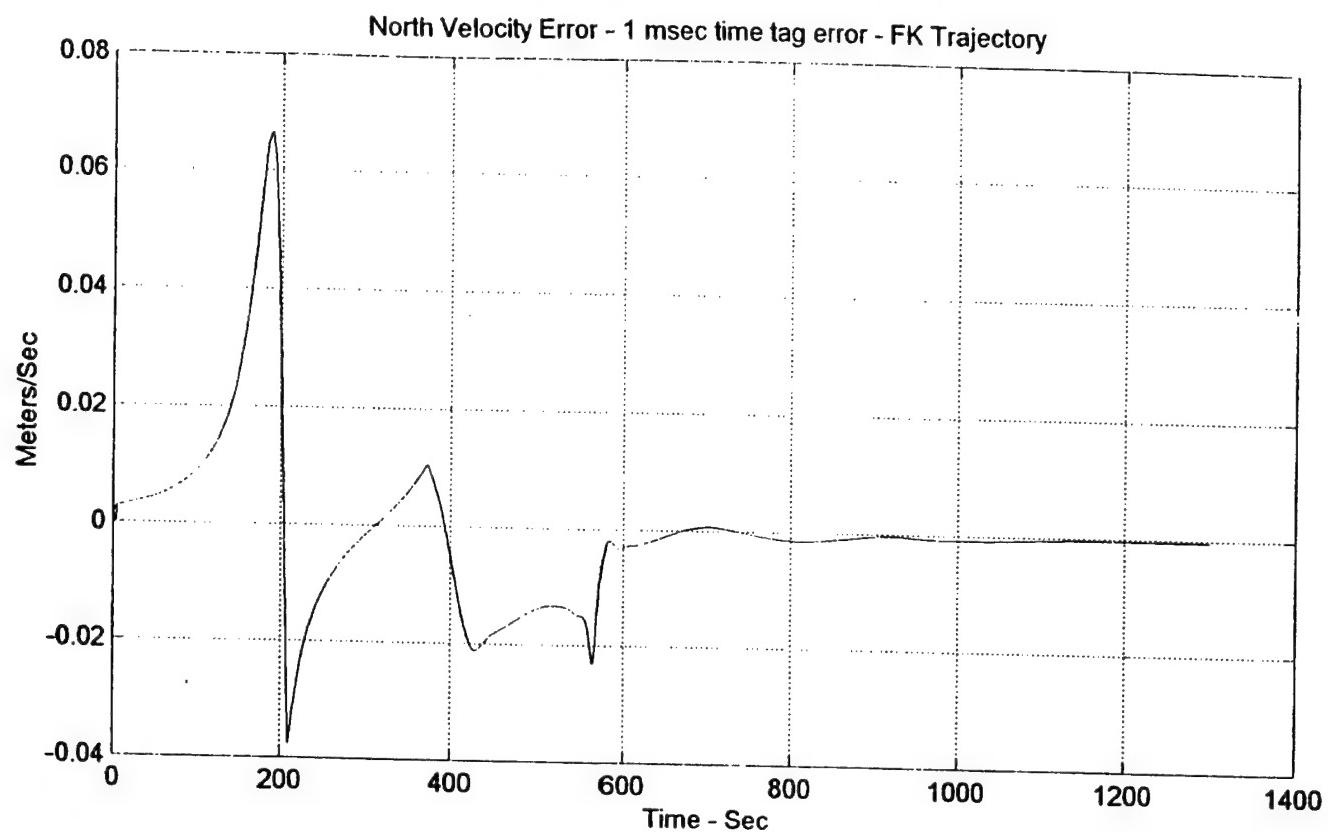


North Position Error - grav errors only - VAND Trajectory(3)

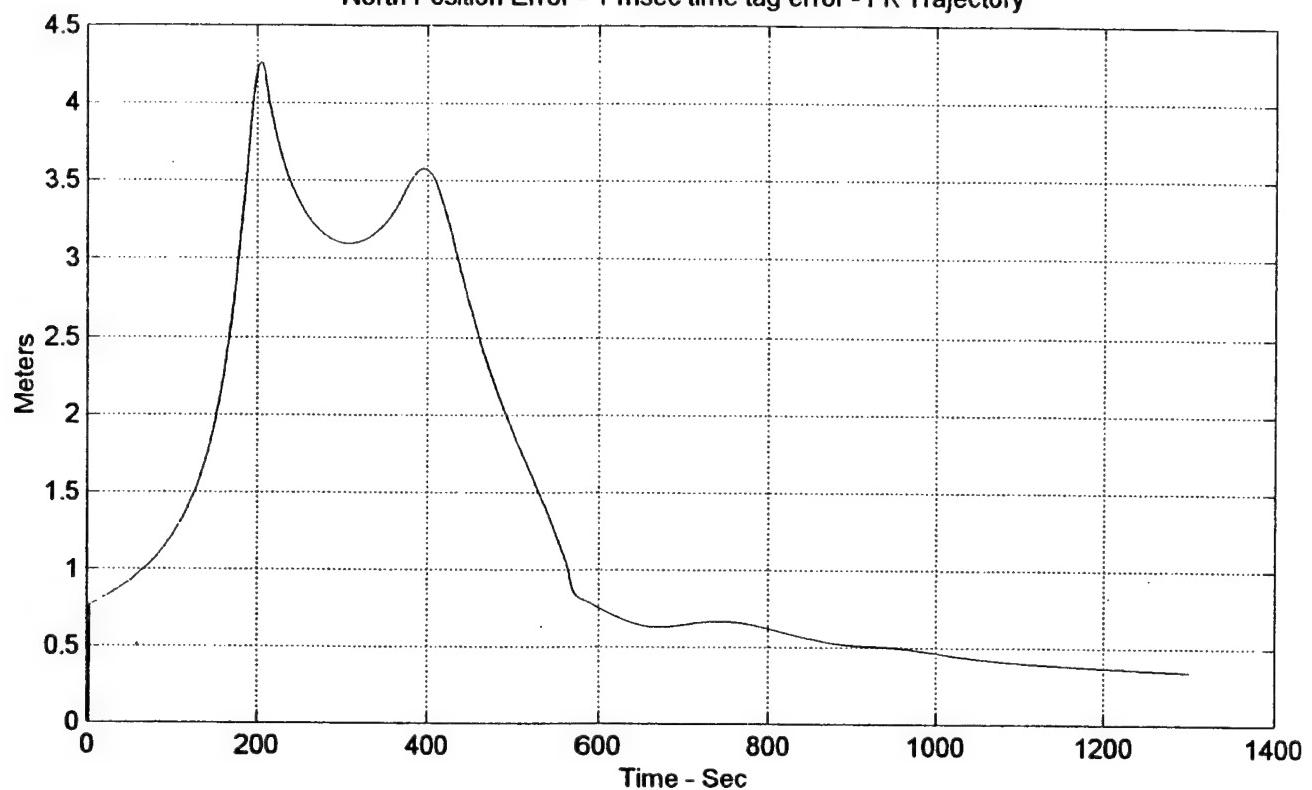


North Position Error - grav errors only (1 micro-g) - FK Trajectory

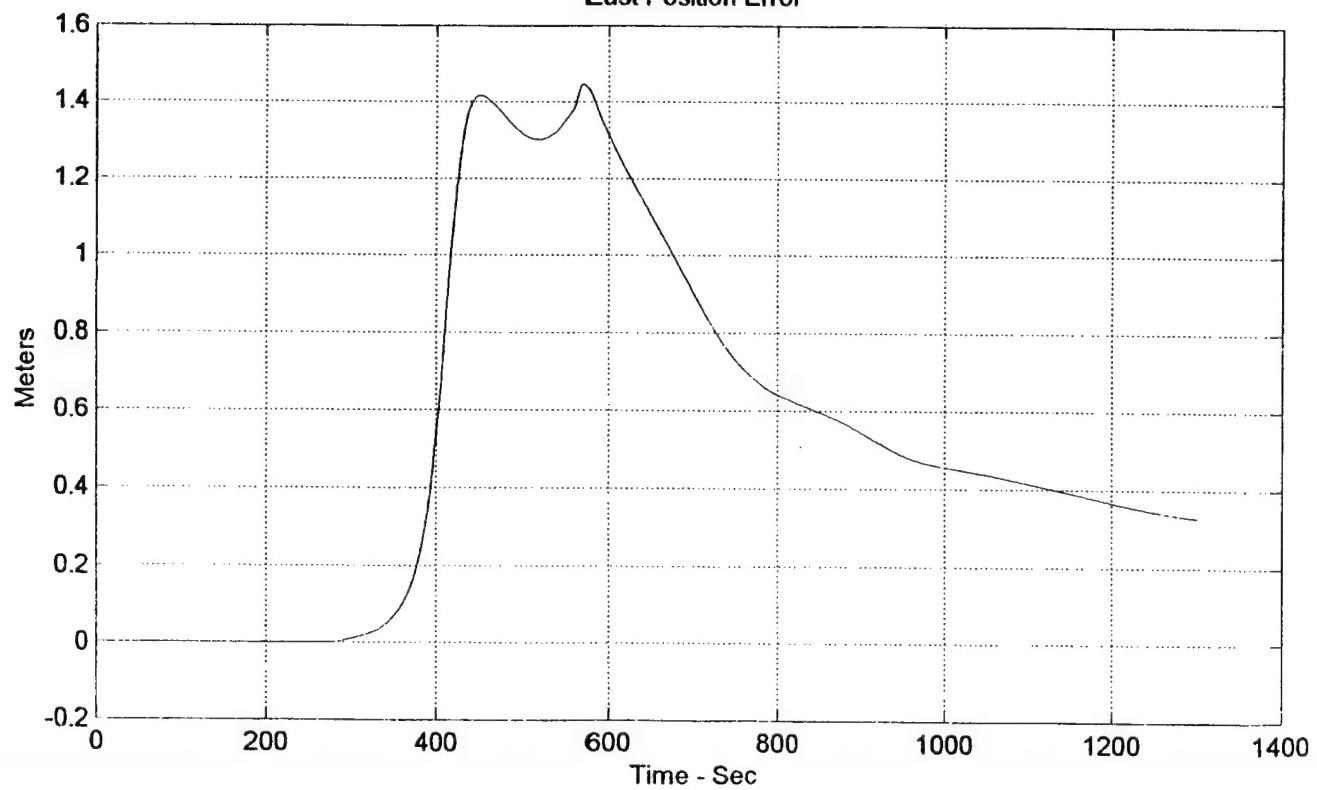


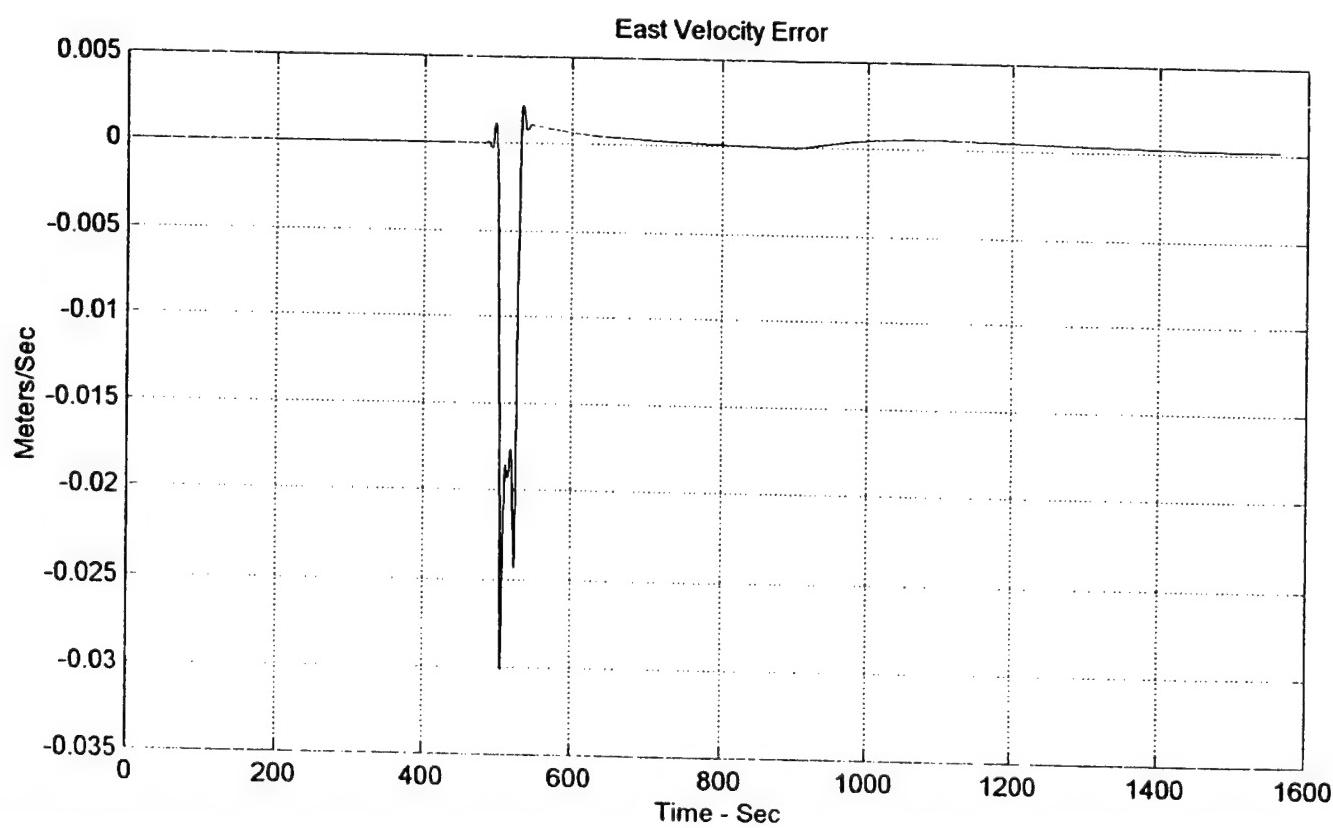
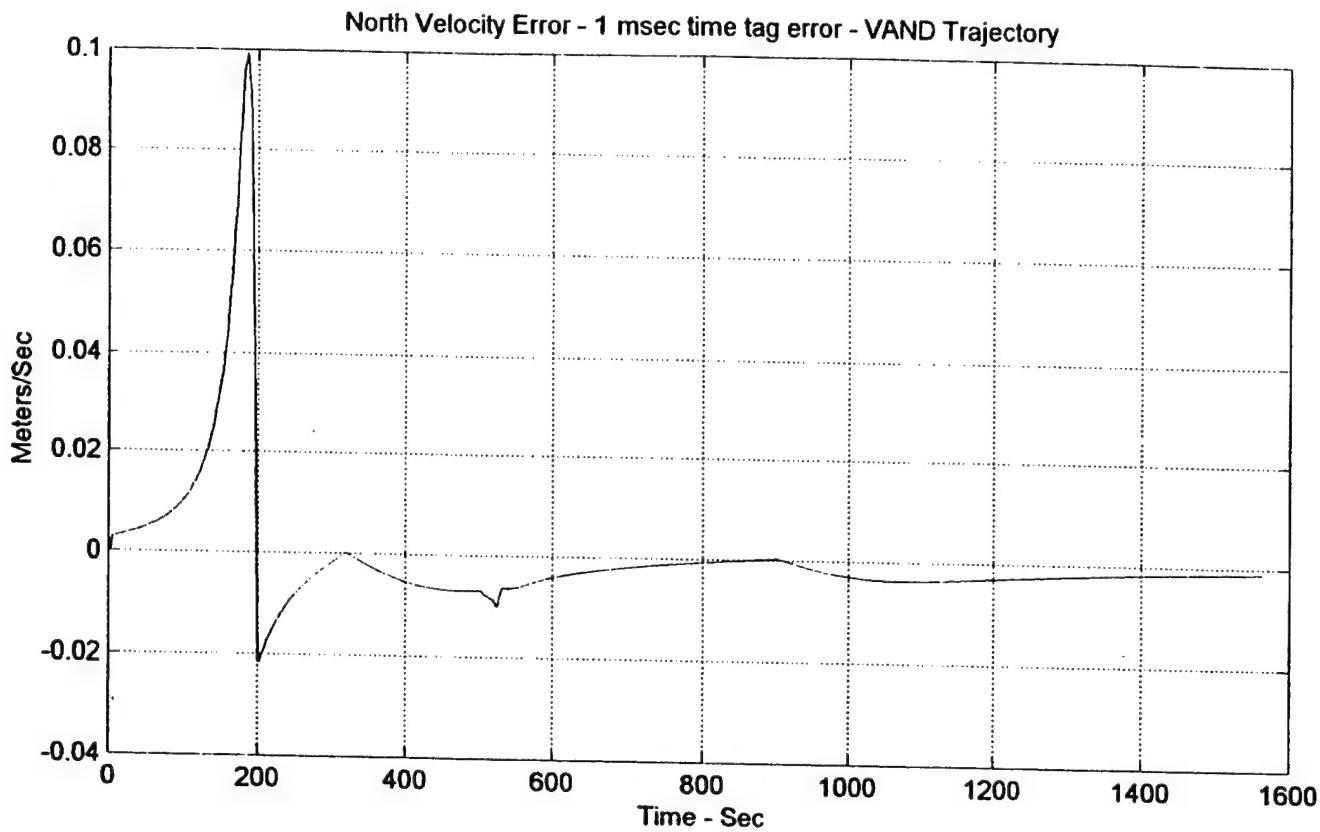


North Position Error - 1 msec time tag error - FK Trajectory

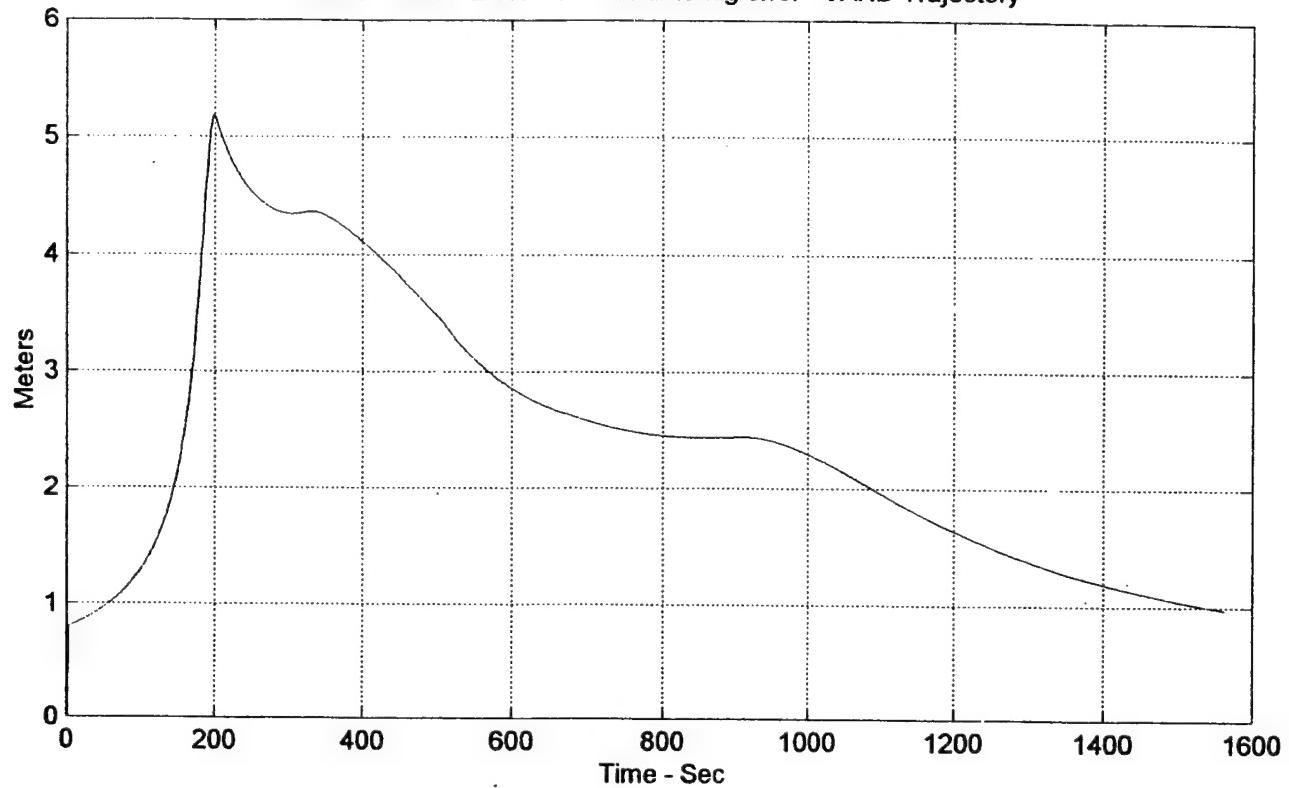


East Position Error

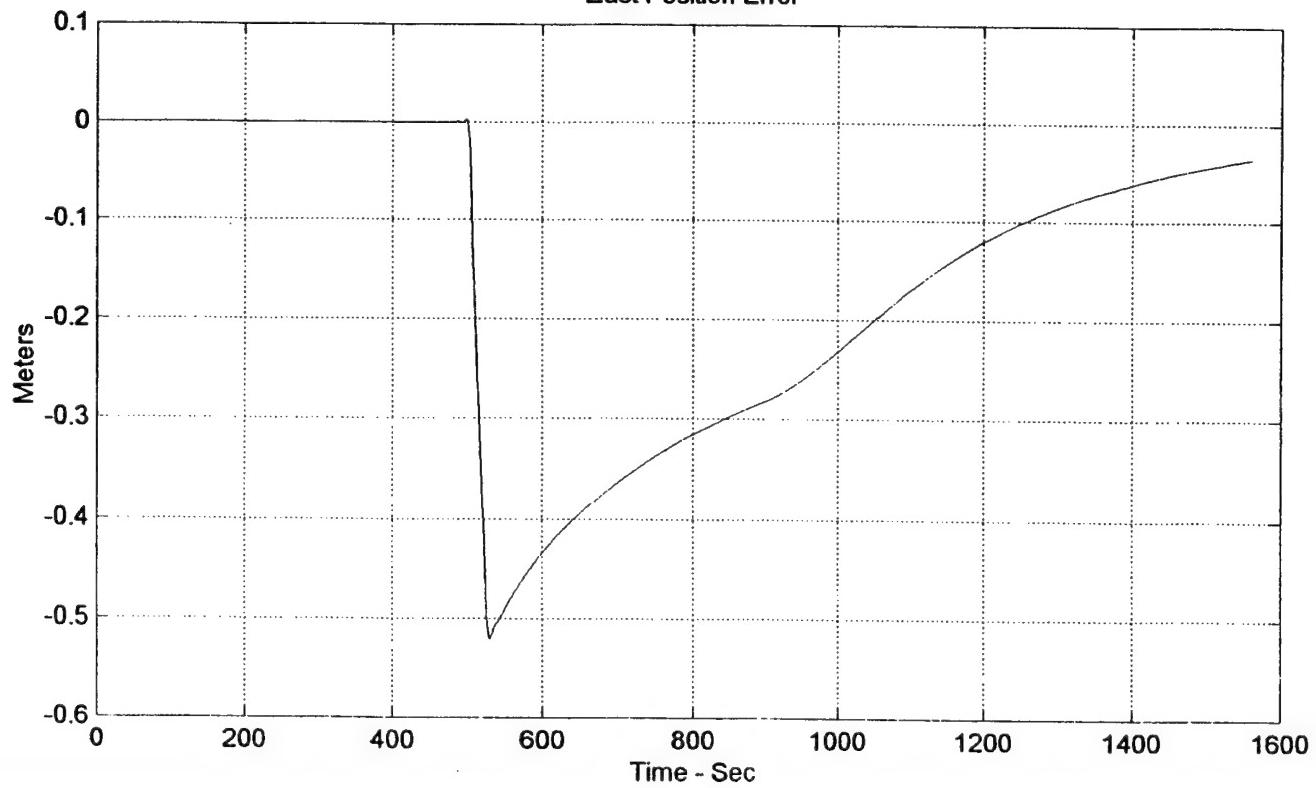




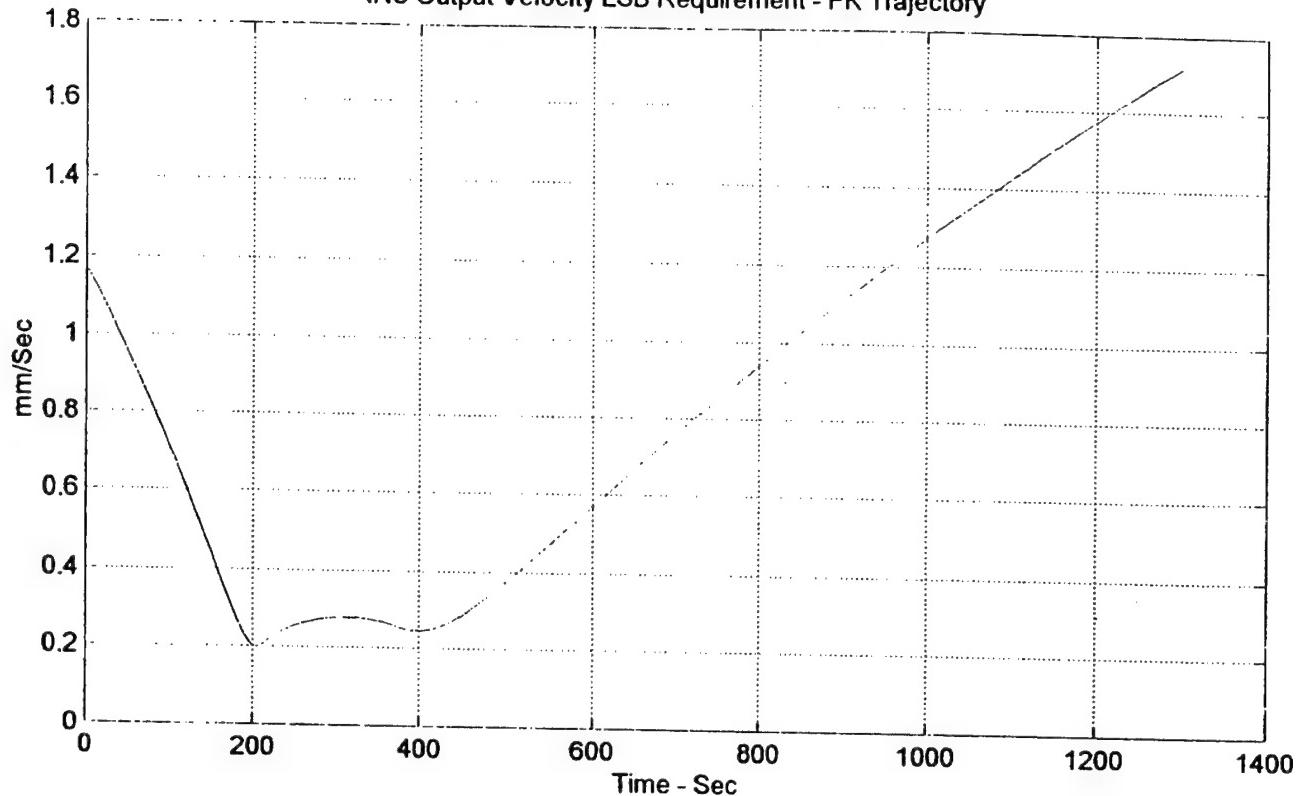
North Position Error - 1 msec time tag error - VAND Trajectory



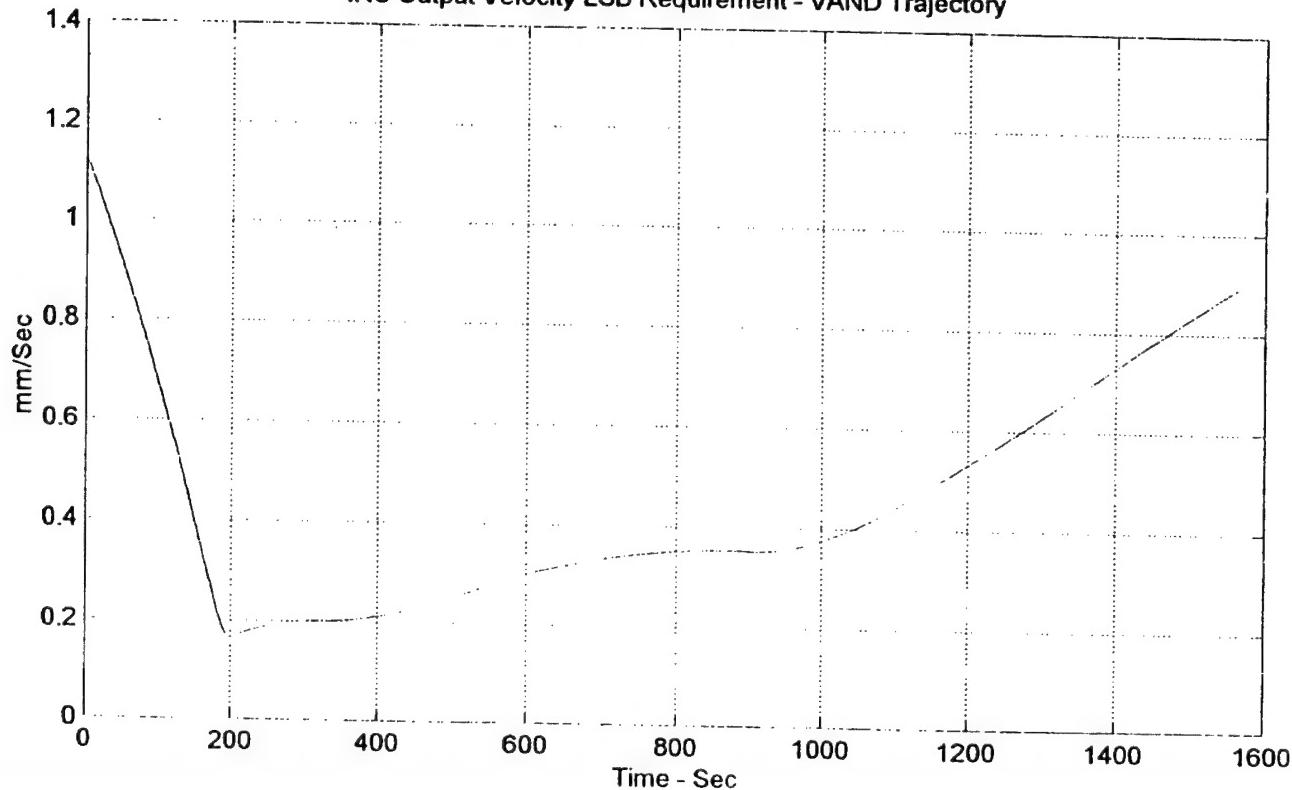
East Position Error



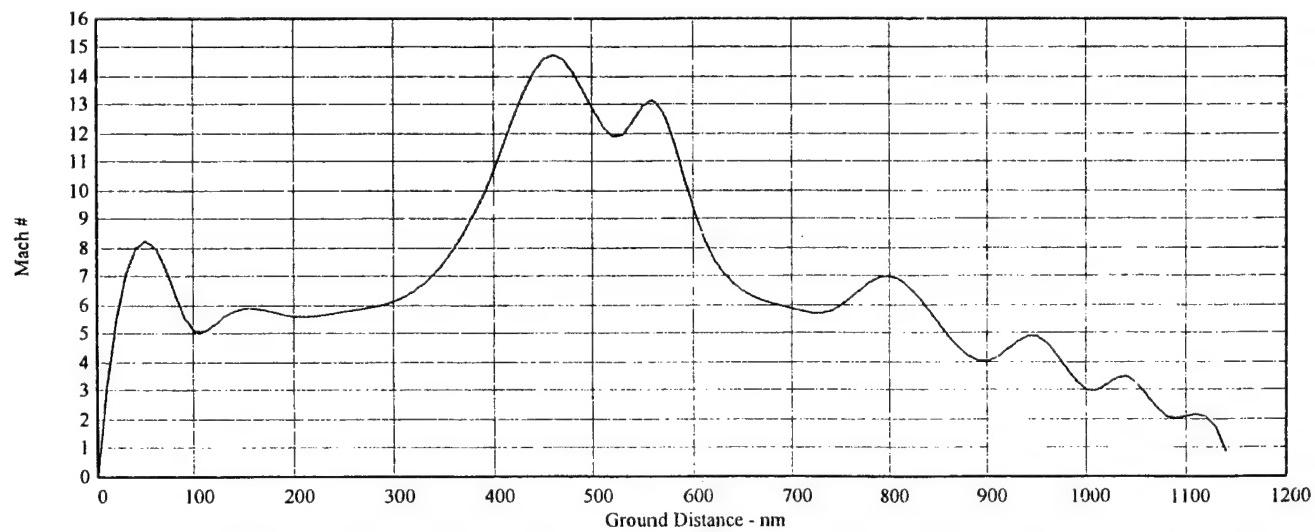
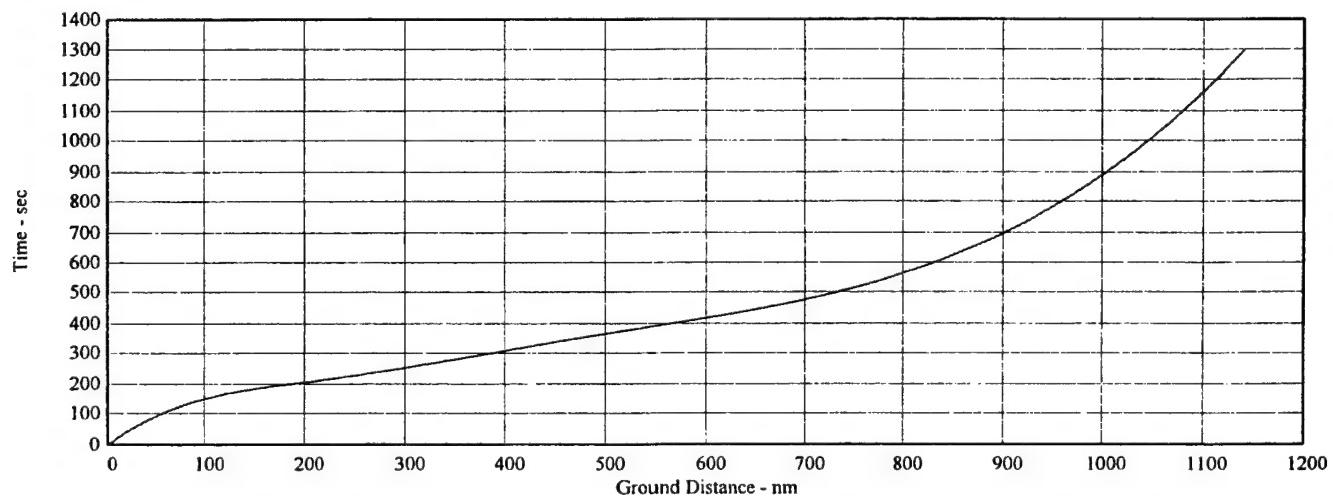
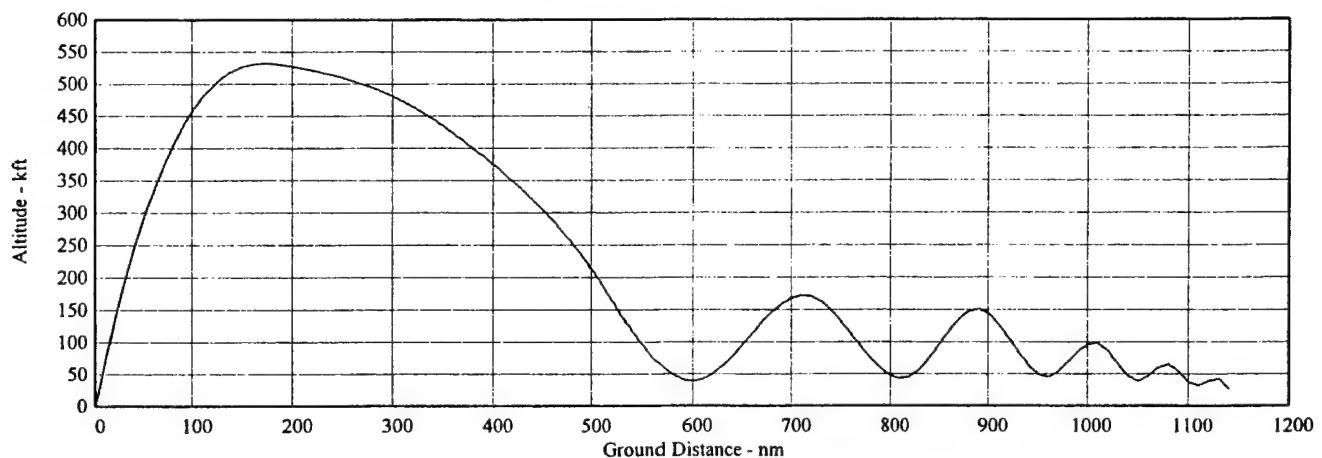
INS Output Velocity LSB Requirement - FK Trajectory



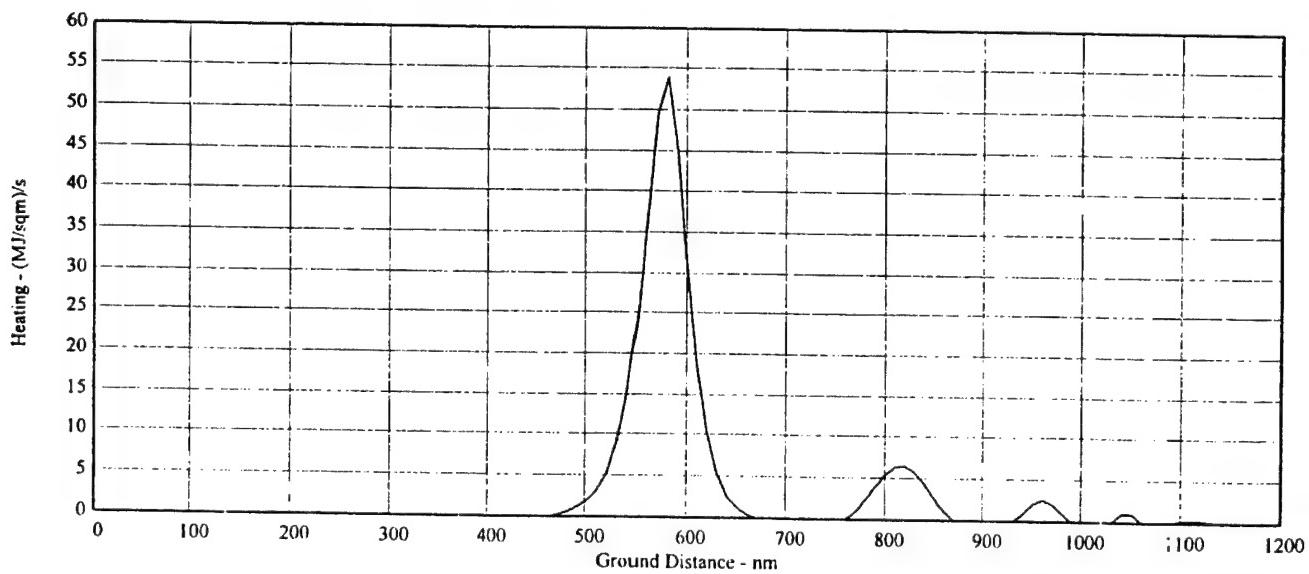
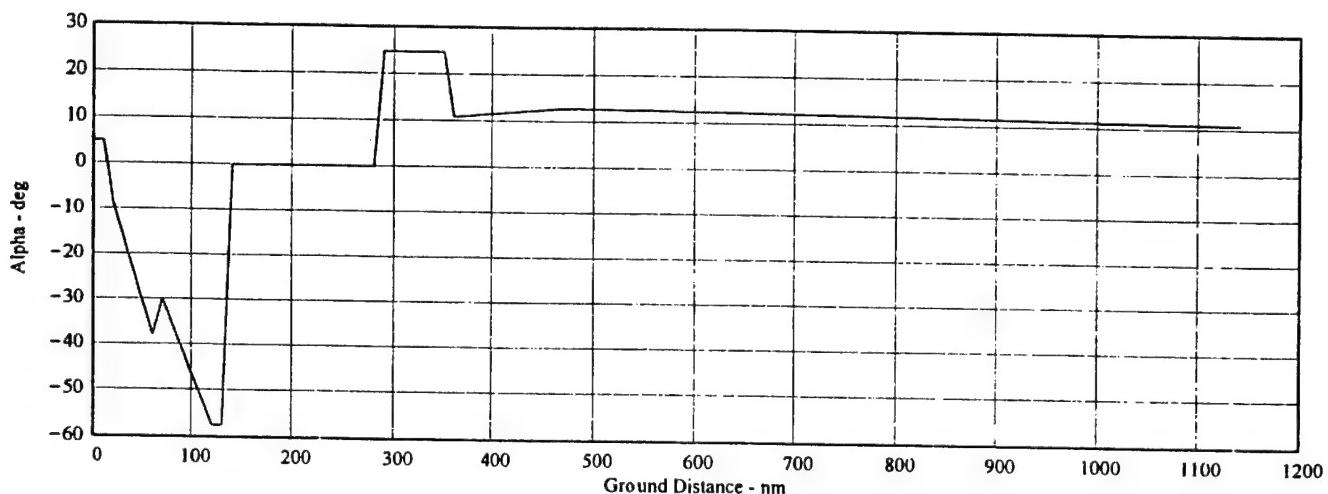
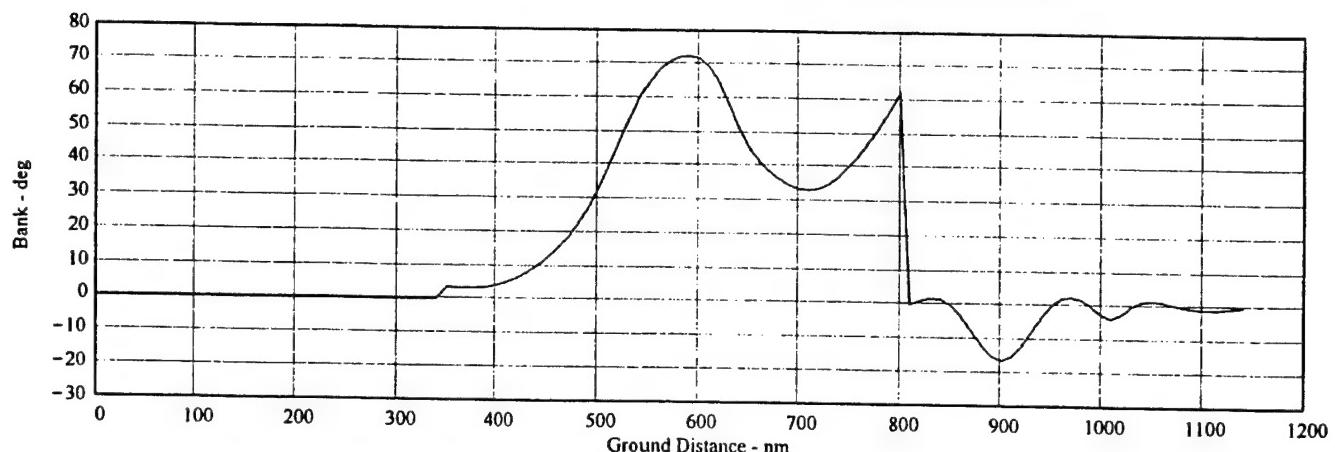
INS Output Velocity LSB Requirement - VAND Trajectory

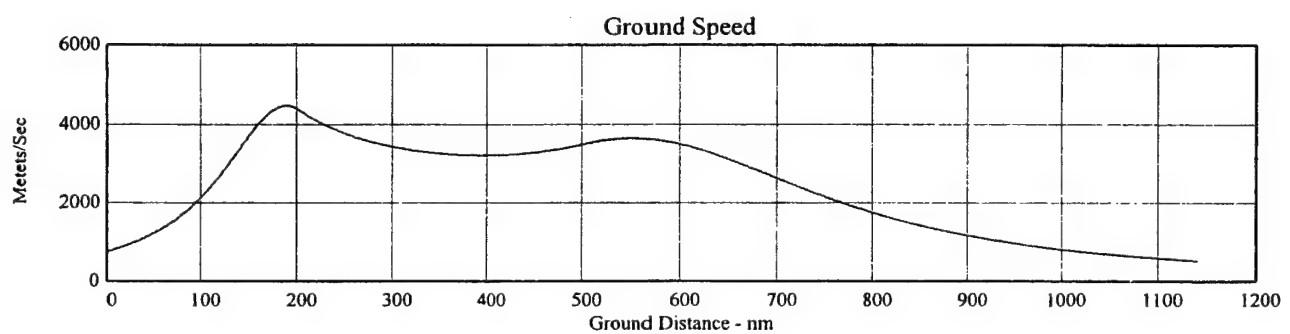
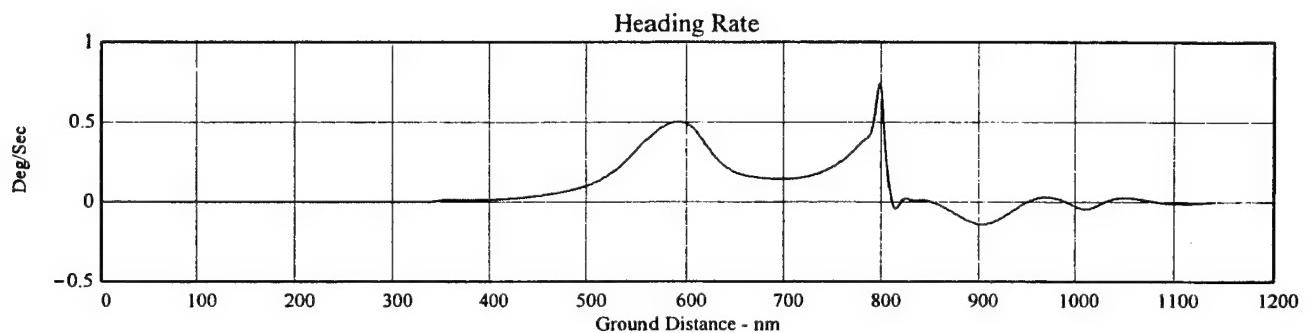
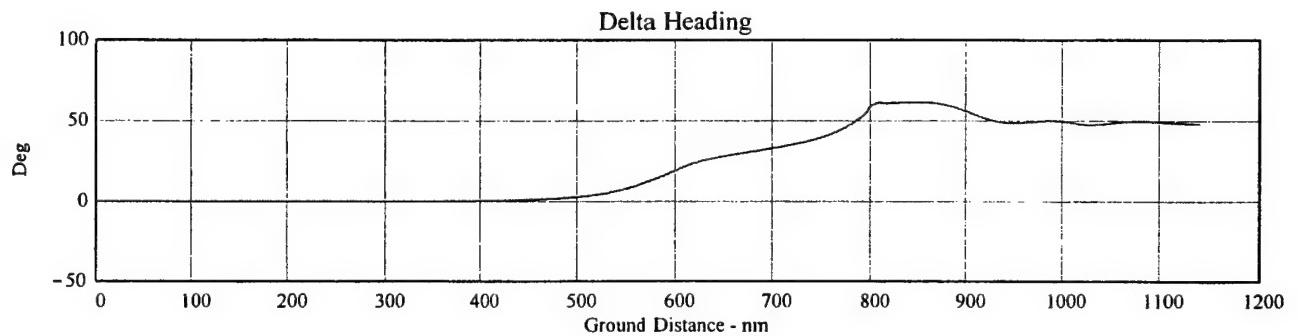


Florida Keys - Trajectory

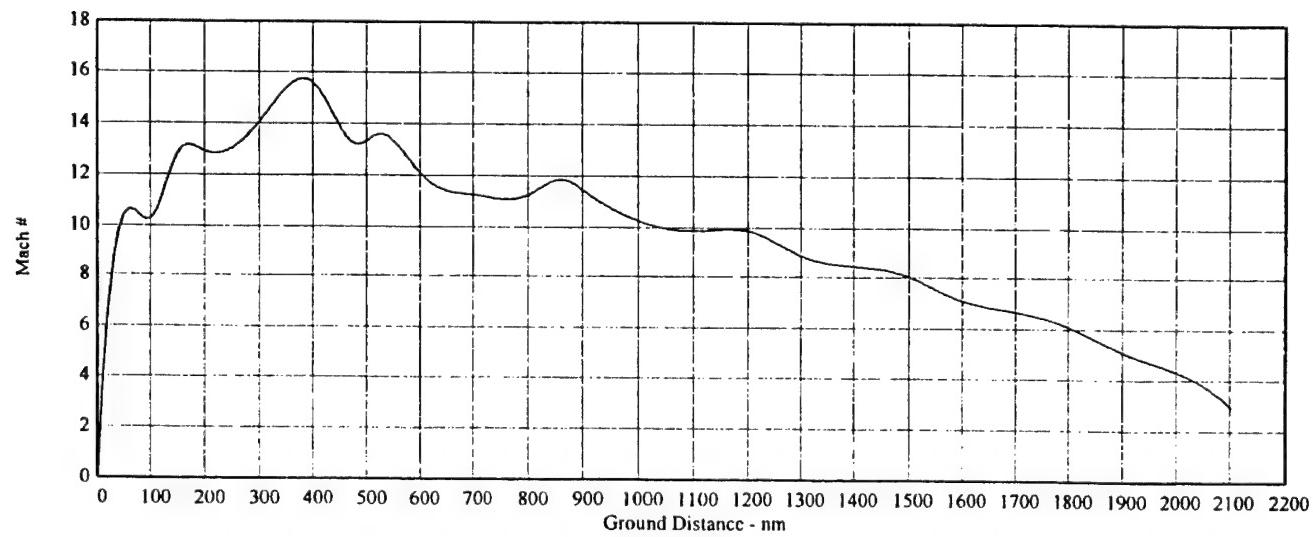
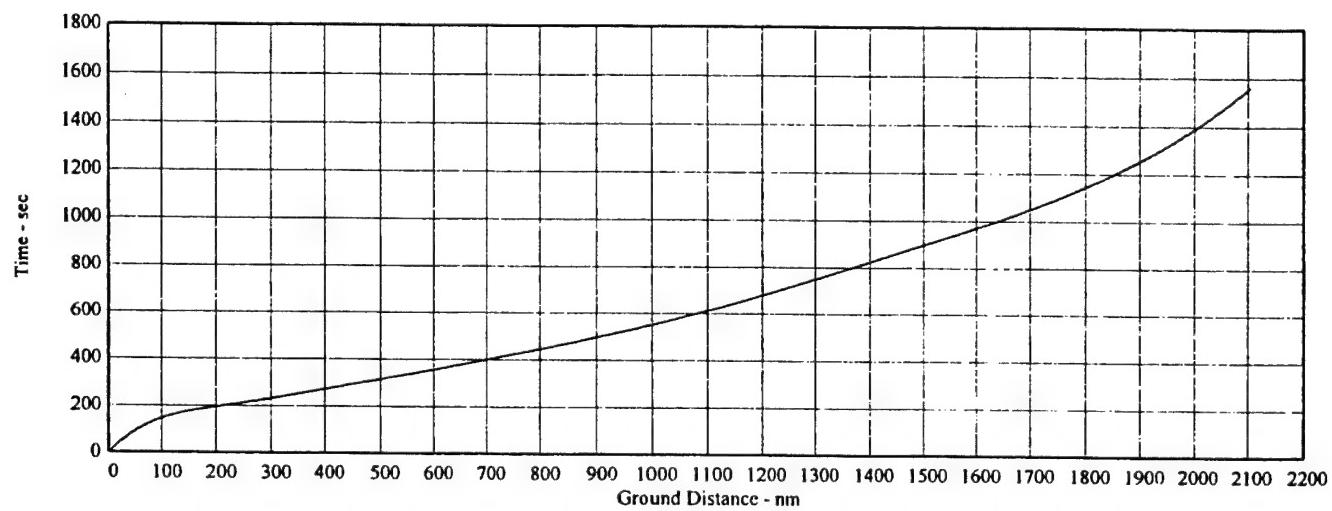
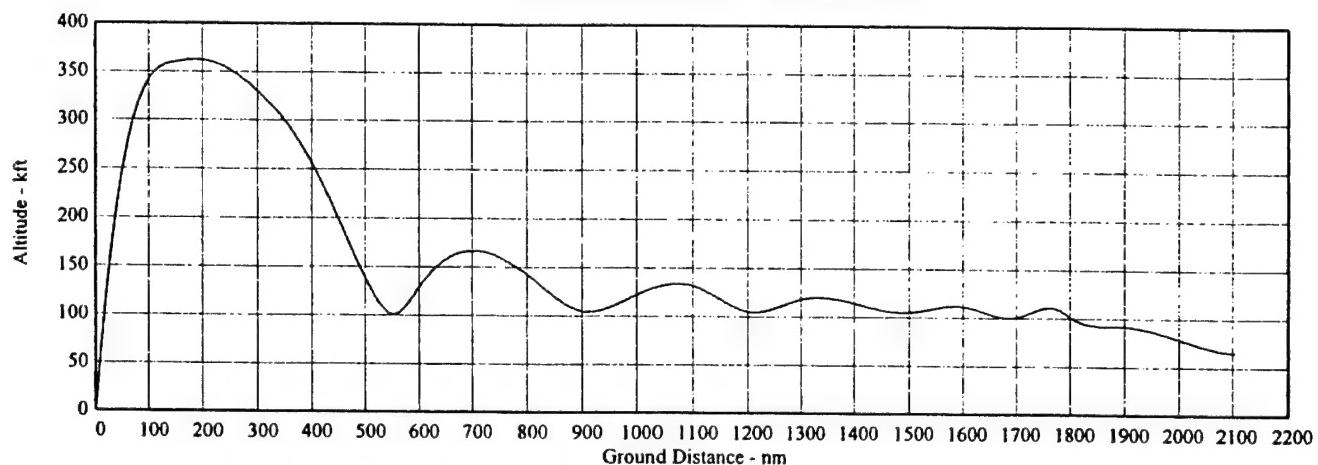


Florida Keys - Trajectory

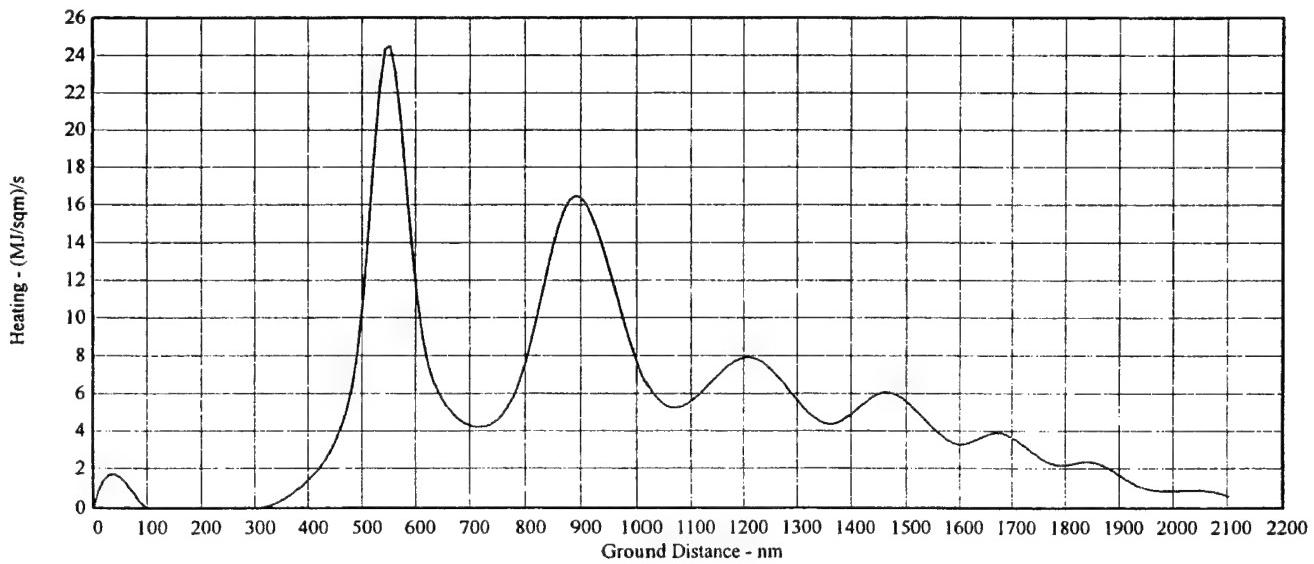
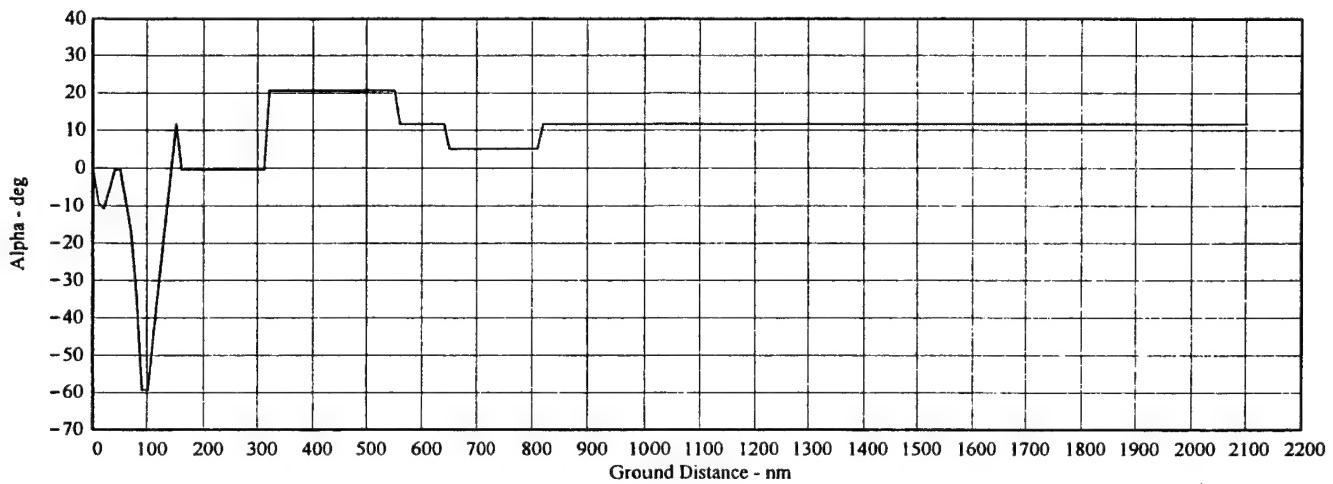
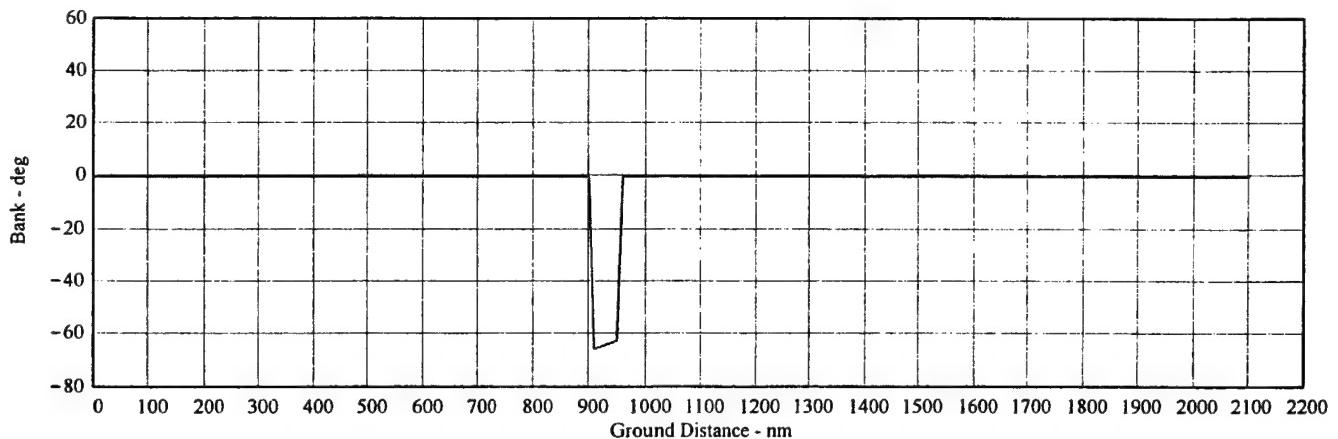


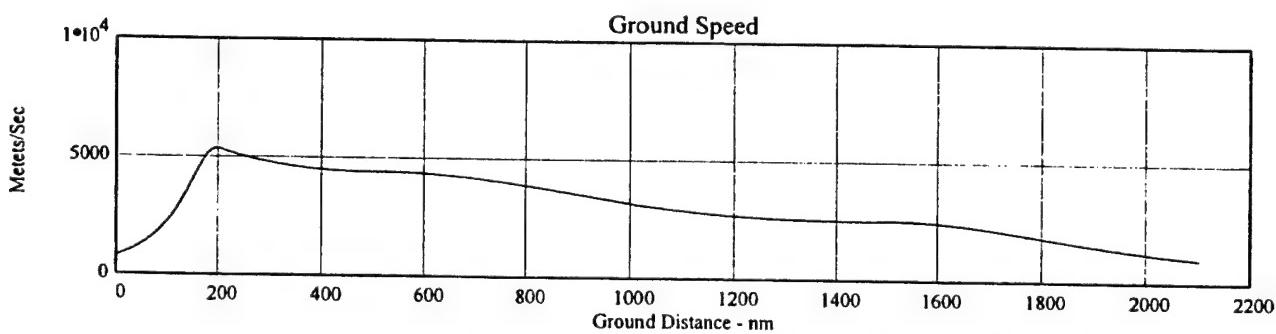
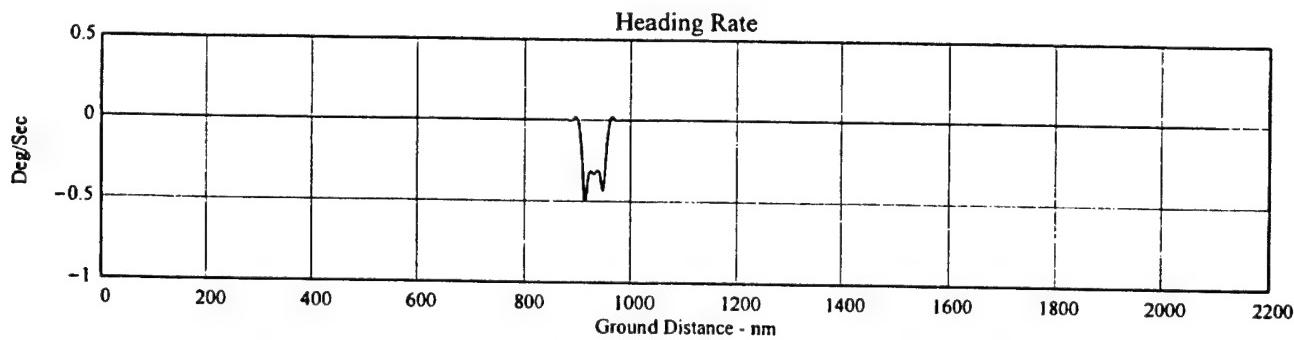
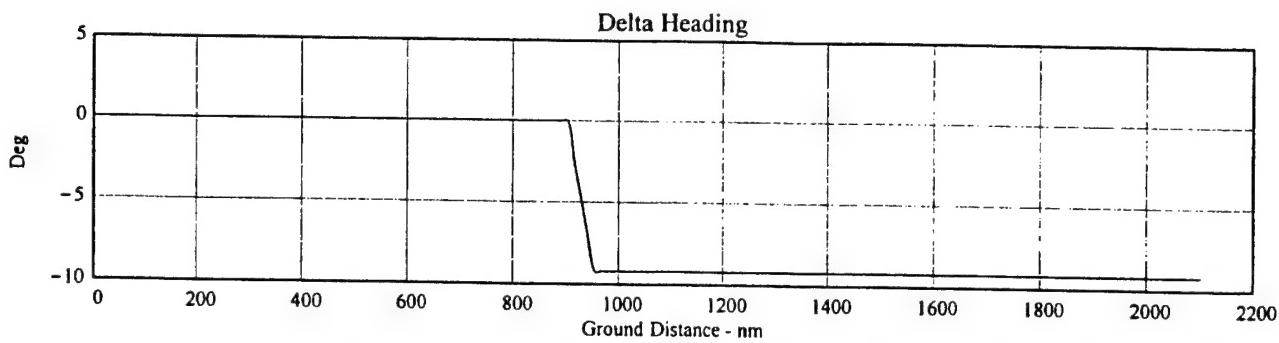


Vandenberg - Trajectory

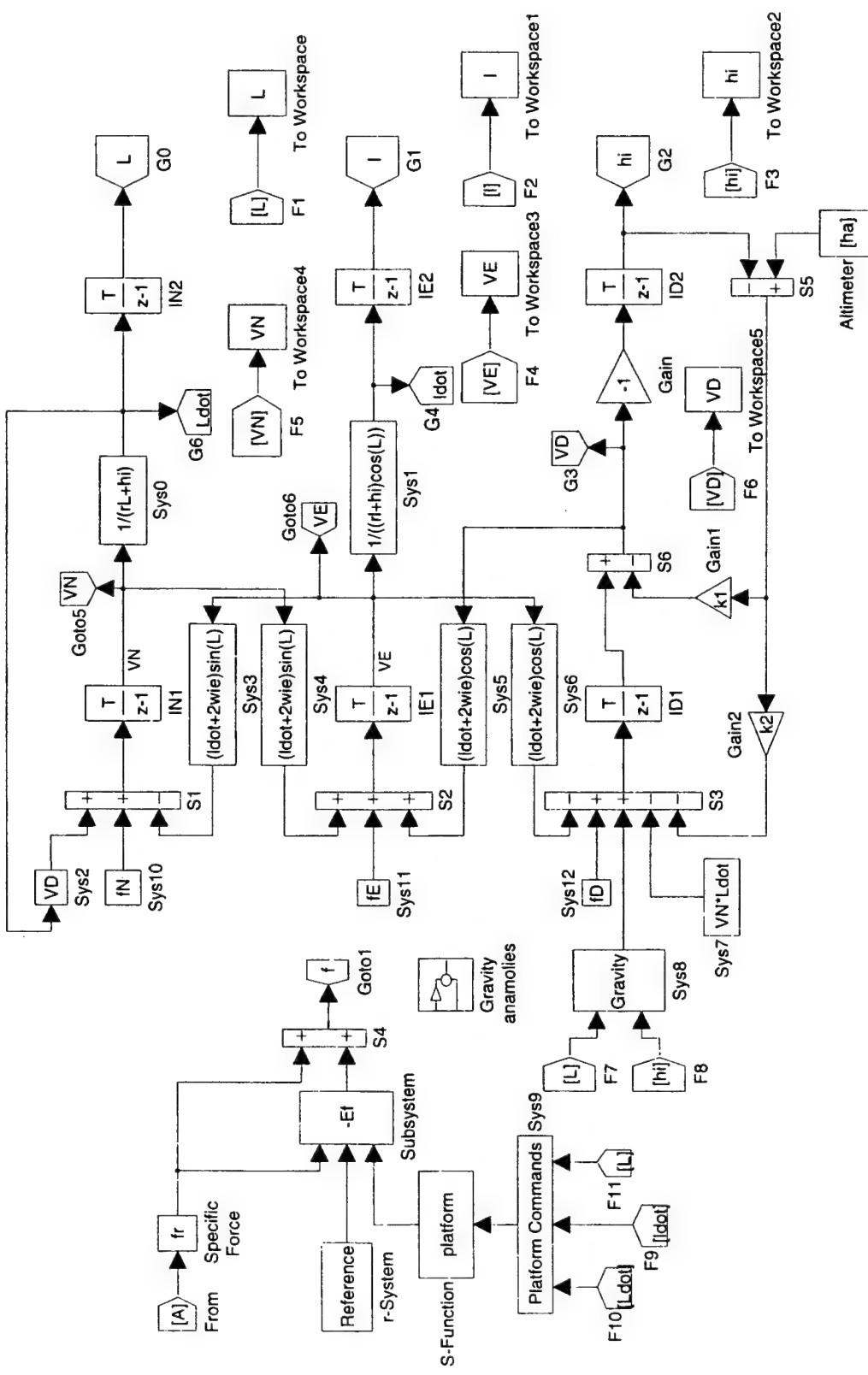


Vandenberg - Trajectory

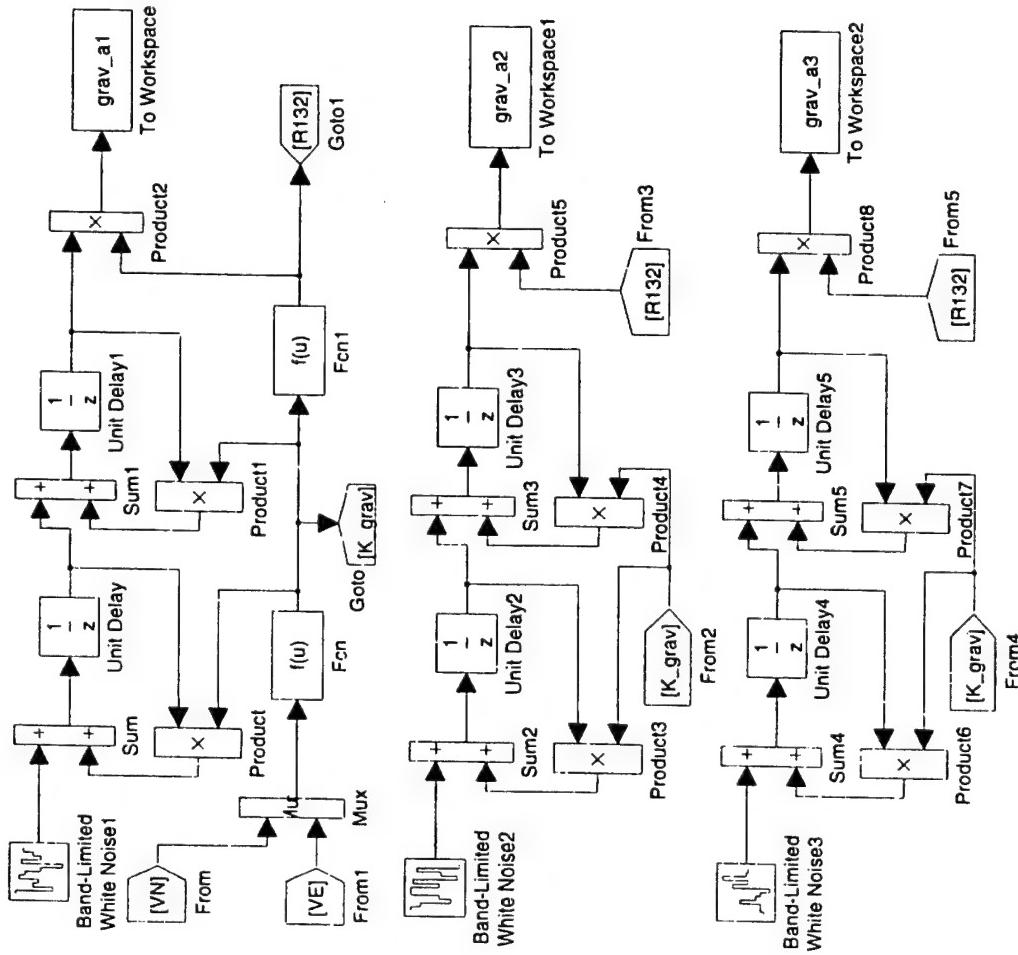




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mg_grav_a/Gravity anomalies

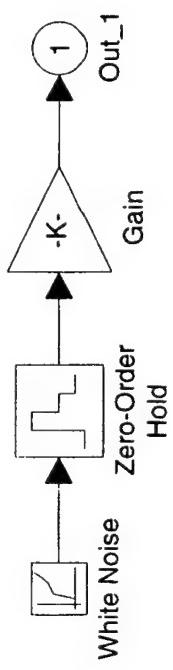


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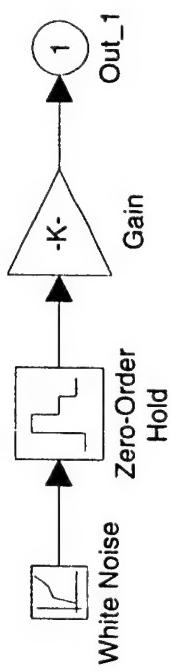


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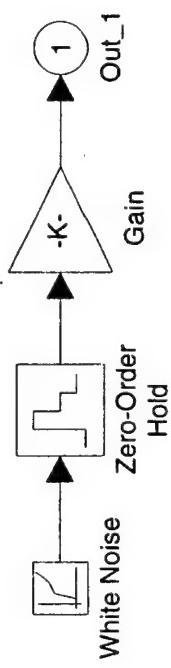


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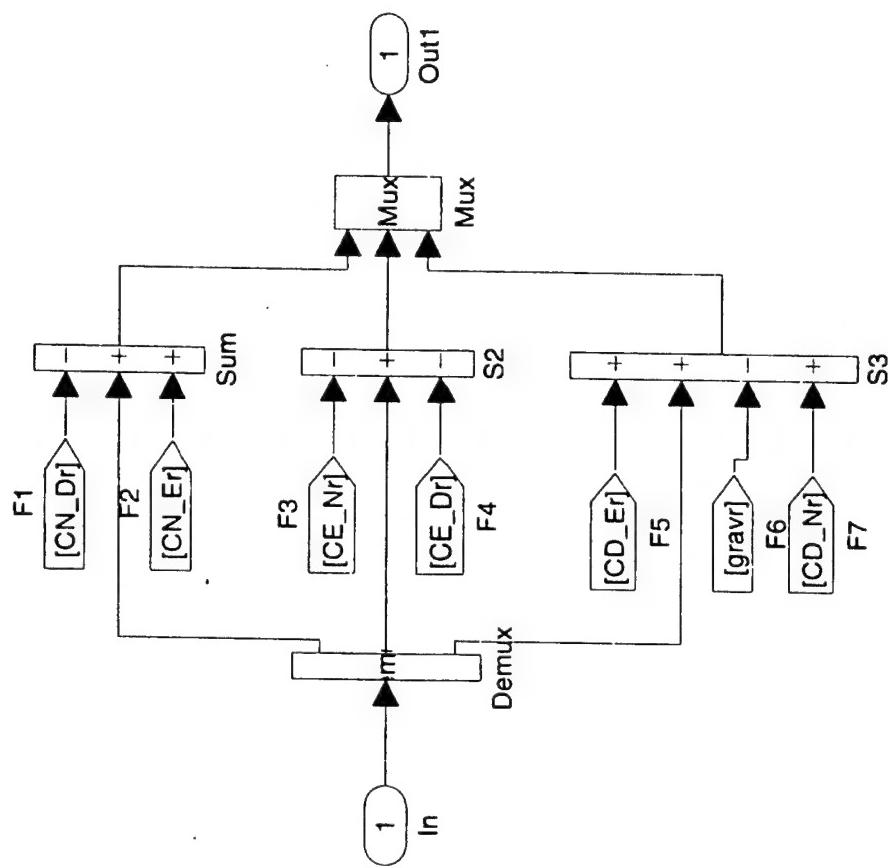


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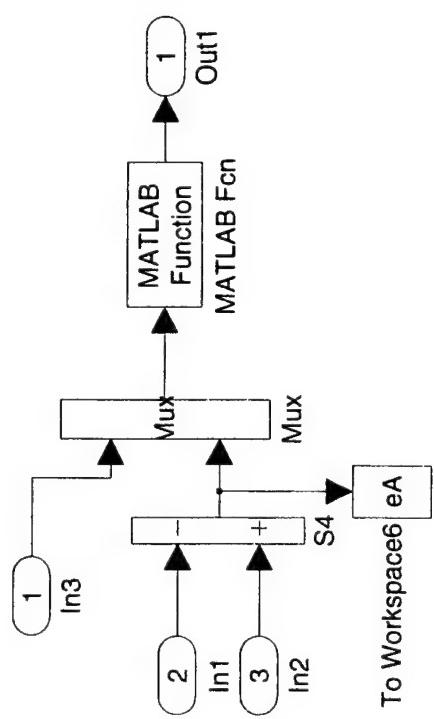


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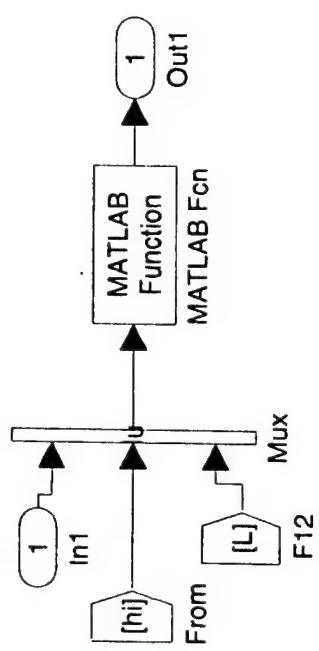
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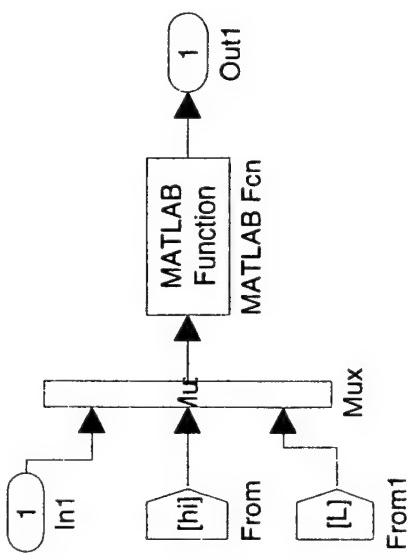
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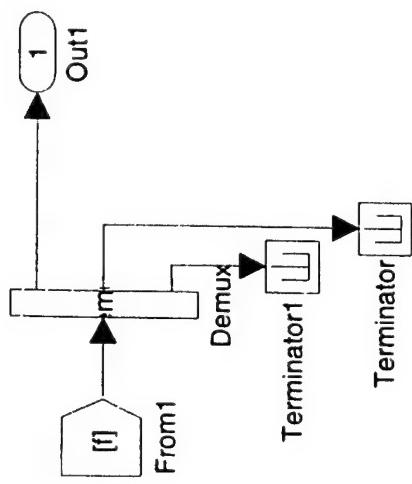
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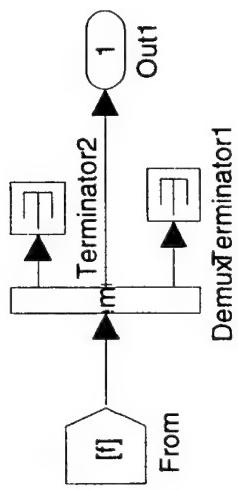


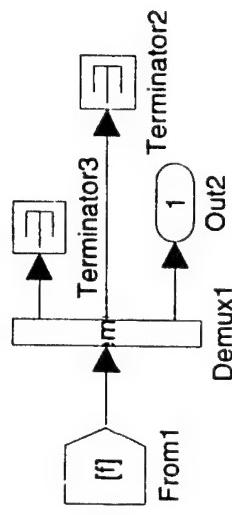
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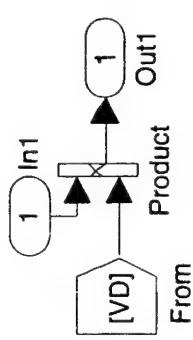


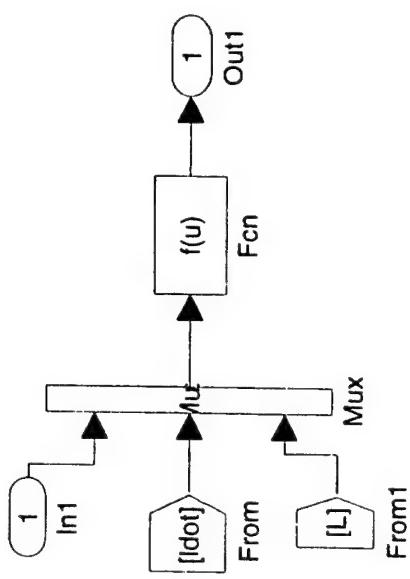


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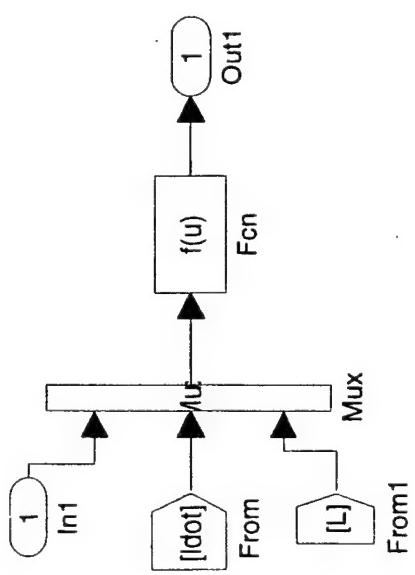


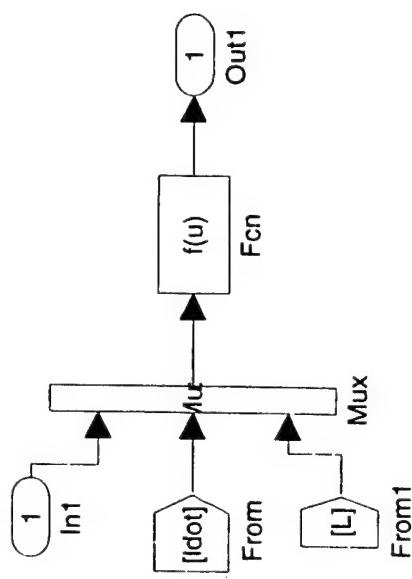


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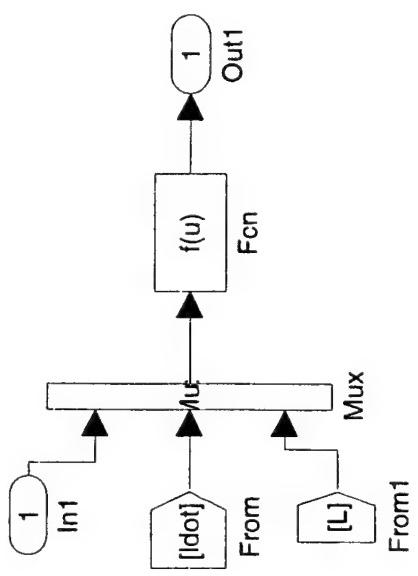
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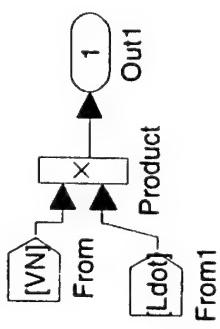
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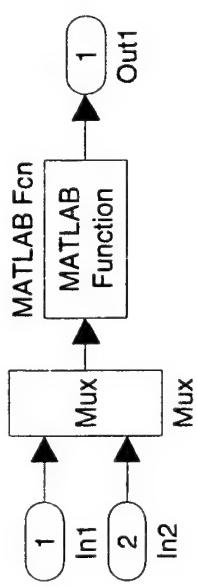




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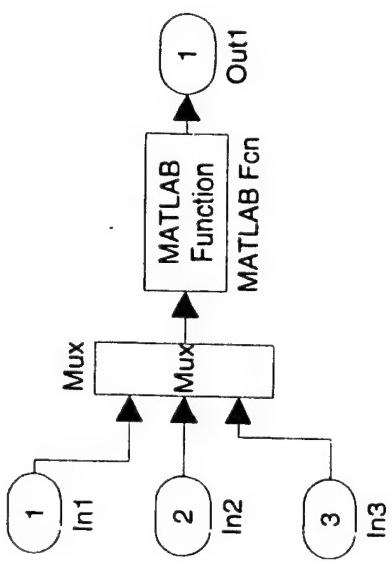
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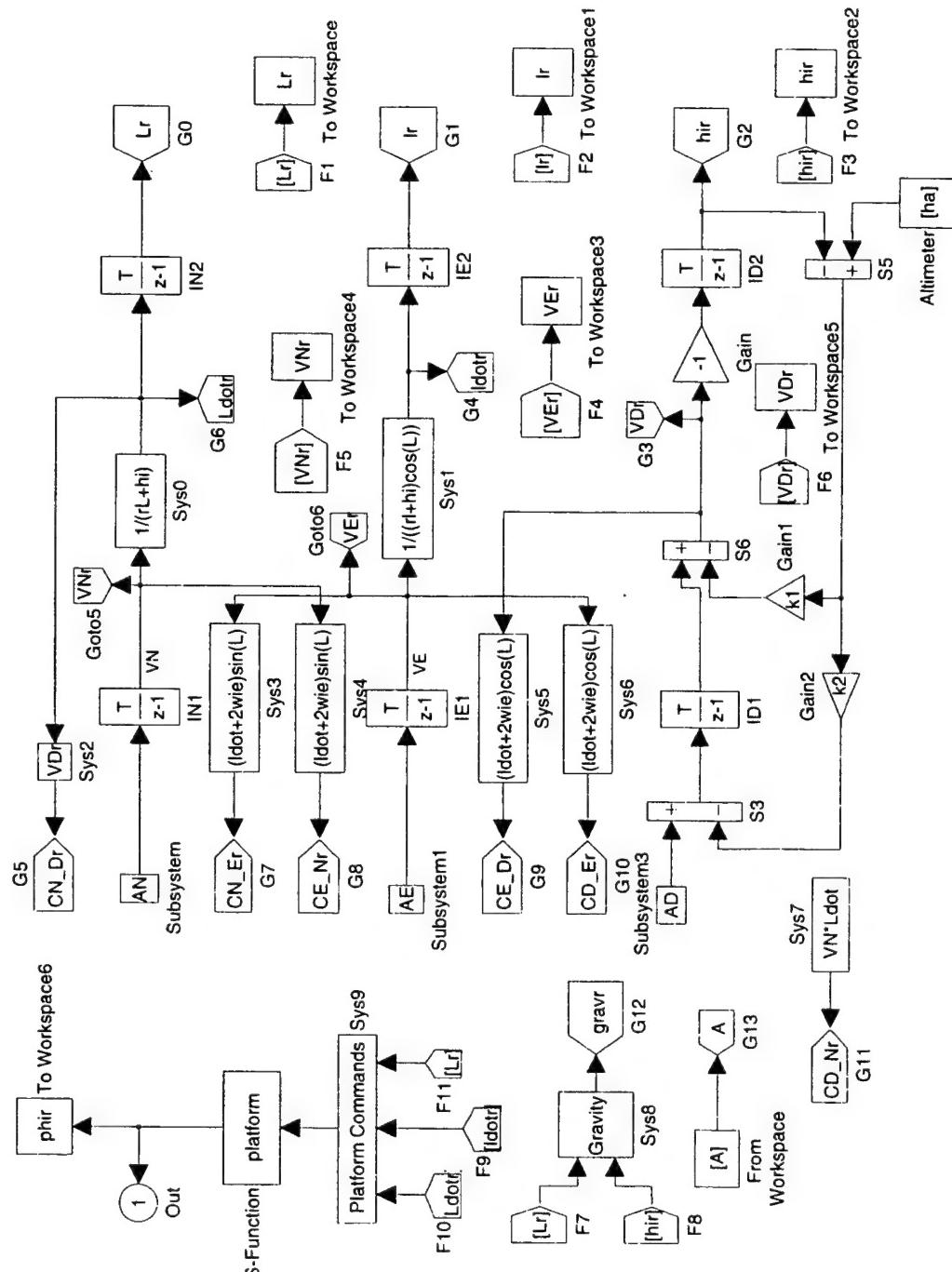


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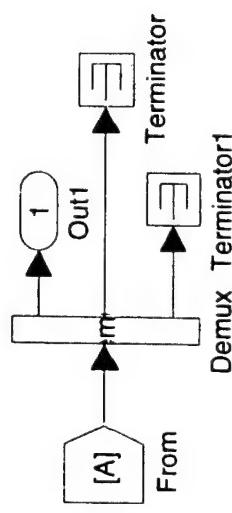


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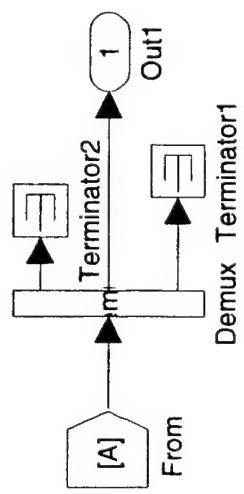


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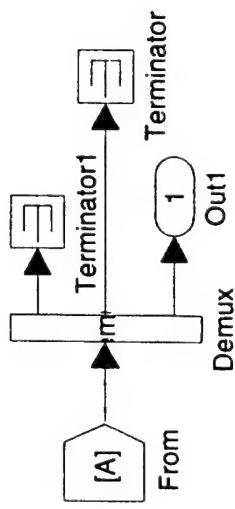
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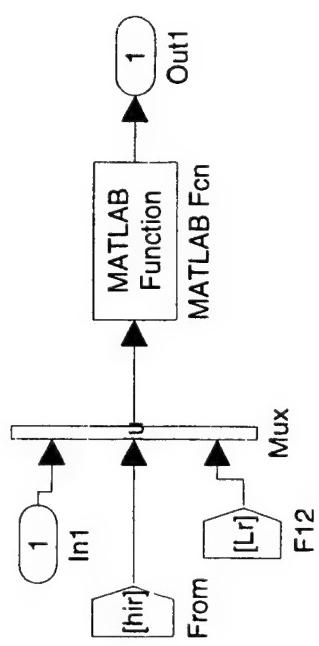


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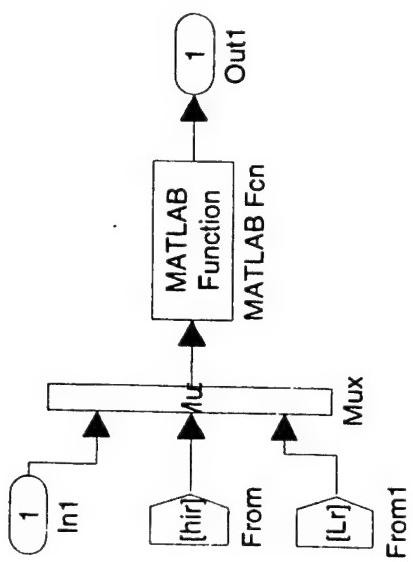


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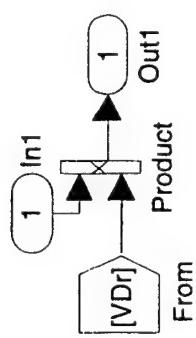


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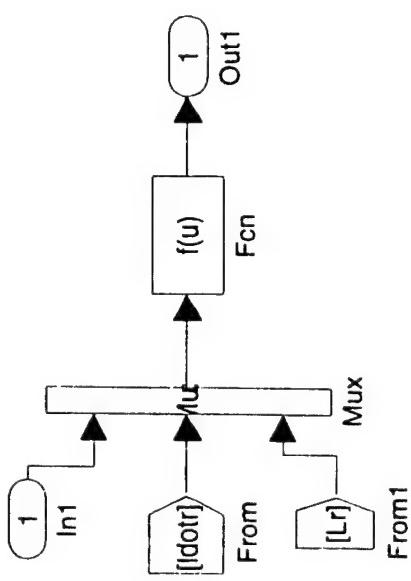


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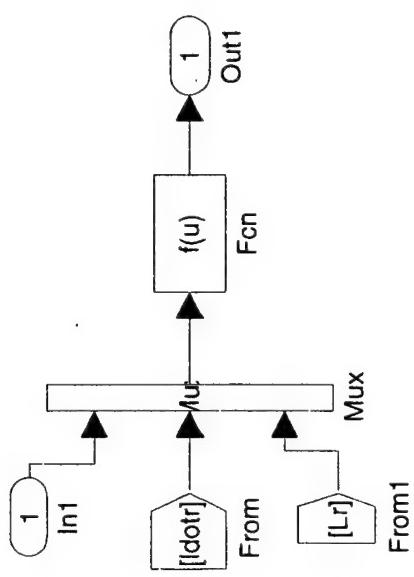
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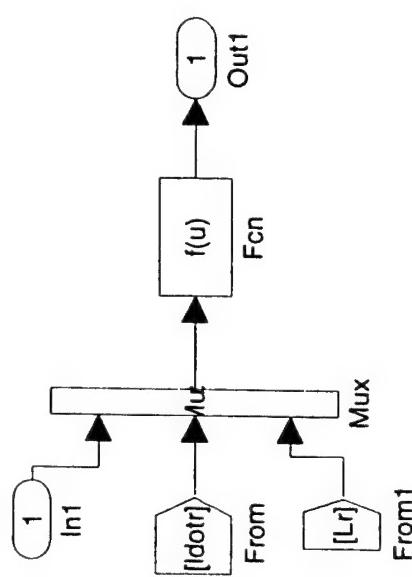
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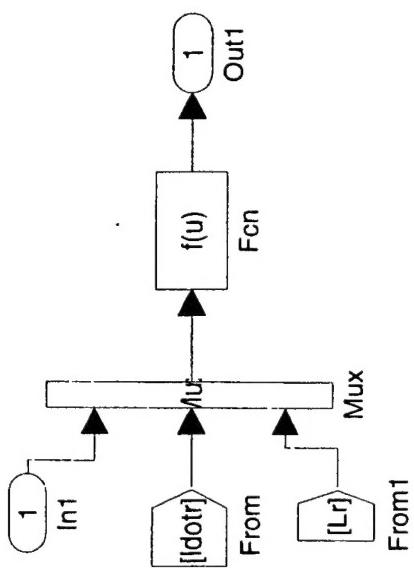
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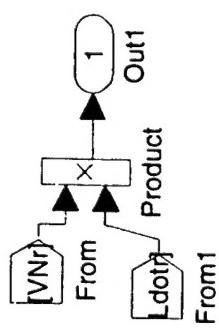


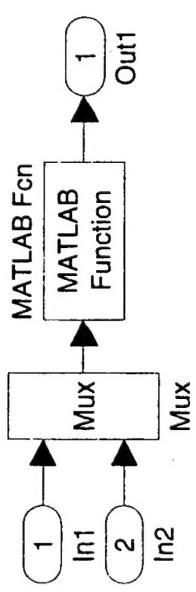
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